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Performance and optimization of pre-stressed beam with respect to shape dimensions

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Motivation

- To prepare simplified numerical codes for evaluation of bending resistance and deflection of a T-beam.
- An extension of simple probability-based model (Le *et al.*, 2018[*]) for resistance of a simply supported pre-stressed concrete beam.
- Idea came from the project related to preparation of advanced precast concrete involving ZPSV copmany and IPM CAS.

(*) Le, D. T. *et al*. (2018) 'Time dependent variation of carrying capacity of prestressed precast beam', *IOP Conference Series: Earth and Environmental Sci.*, 143(2018) 012013, IOP Publishing, doi:10.1088/1755-1315/143/1/012013.

Outline

- Introduction
- Material properties
- T-beam bending resistance and deflection
- Probabilistic modelling
- Numerical examples
- Sample section optimization
- Summary
- Conclusions

Introduction

- Goals:
  - Set up numerical procedures for evaluation of bending resistance and deflection of the T-beam.
  - Consideration of random variation of time-dependent concrete strength and elastic modulus.
  - Assess performance of pre-stressed concrete beam with respect to section optimization.

Introduction (cont.)

- *Methodology:*
  - Set up a simple model for the probability-based bending resistance and deflection computation of a simply supported pre-stressed concrete T-beam.
  - Prepare a procedure for the estimation of time dependent bending resistance and deflection of the beam using Matlab environment.

Introduction (cont.)

- ***Factors to be considered:***
  - Availability of time dependent behavior of basic properties of concrete:
    - Elastic modulus
    - Compressive strength
  - Position of pre-stressing tendons in the section.

Introduction (cont.)

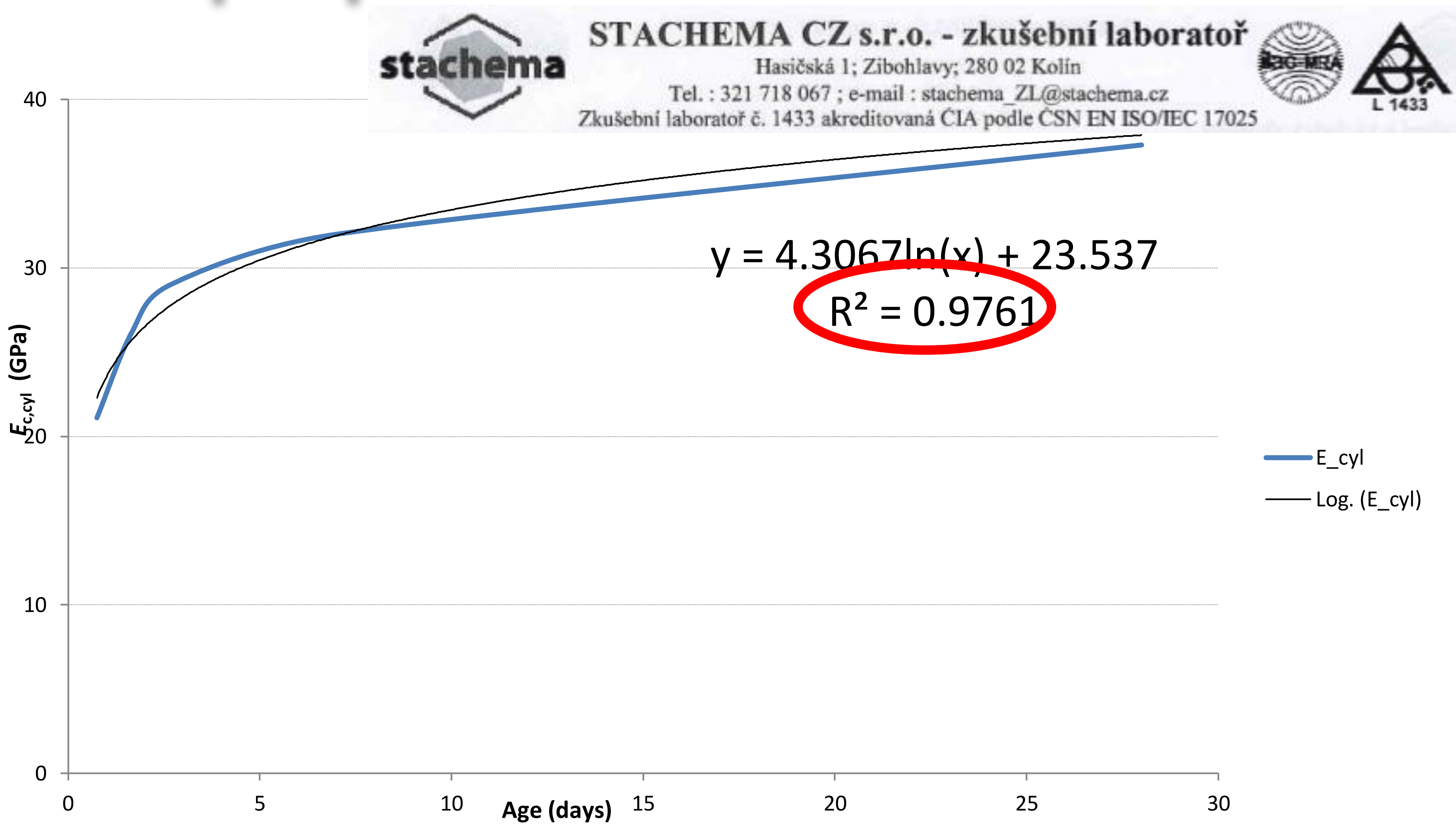
- ***Used means:***

  + Monte Carlo simulation based approach.

  + Normal distributions of compressive strength and elastic modulus of concrete as well as effective height of the cross section.

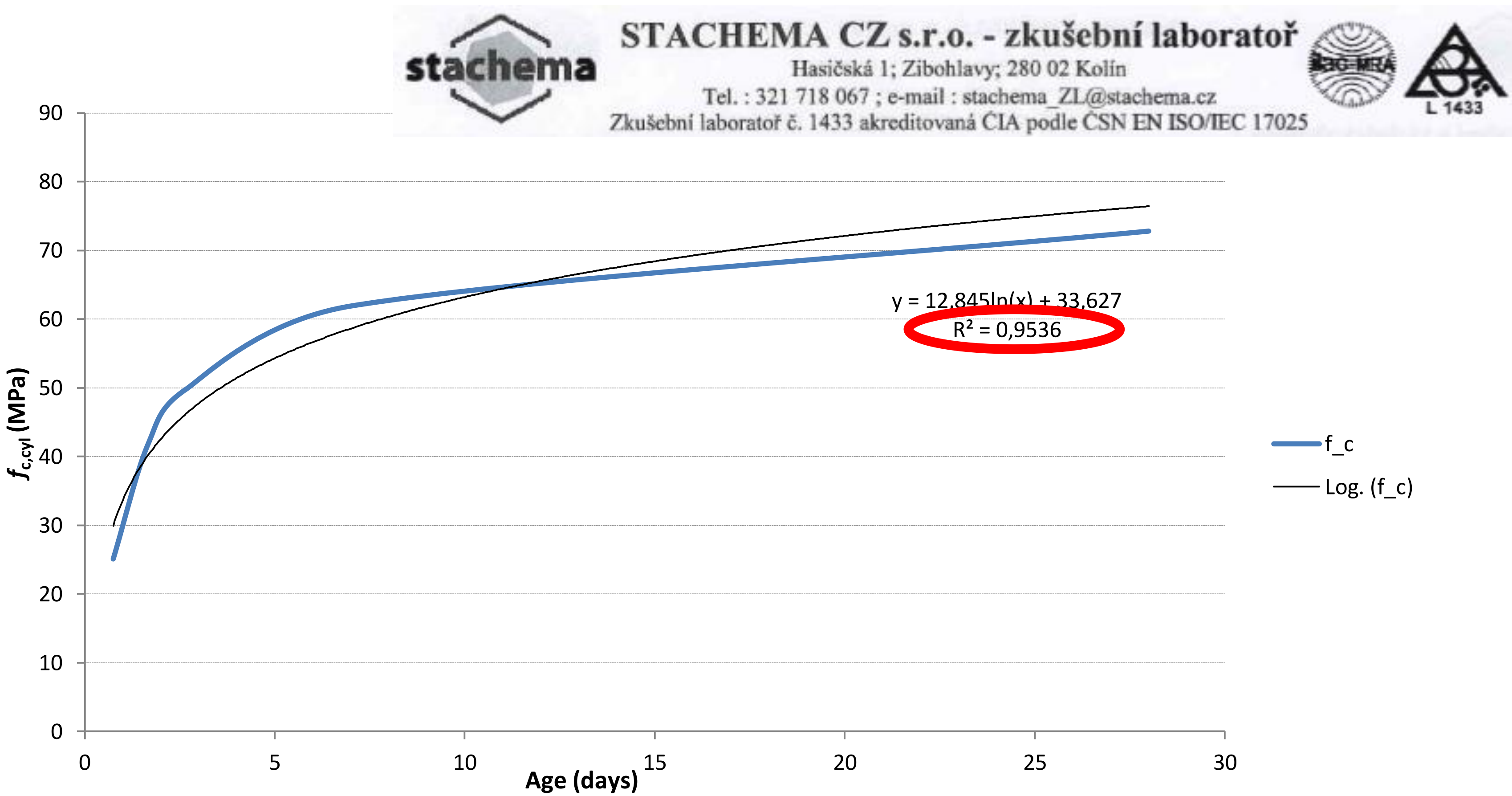
  + Matlab compatible environment.

Material properties



Approximation of considered elastic modulus time dependent behavior (C50/60): $E_{c,cyl}(t) = 4.3067\ln(t) + 23.537$.

Material properties (cont.)



Approximation of considered cylinder strength time dependent behavior (C50/60):

$$f_{c,cyl}(t) = 12.845\ln(t) + 33.627$$

Material properties (cont.)

Elastic modulus of concrete sample at *t* days :

$$E_{\mathrm{c,cyl}}(t) = \mu\left(E_{\mathrm{c,cyl}}(t)\right) + cov\left(E_{\mathrm{c,cube}}(28)\right).\mu\left(E_{\mathrm{c,cyl}}(t)\right)$$

$\mu\left(E_{\mathrm{c,cyl}}(t)\right)$: mean value of elastic modulus of sample at age *t*;

$cov\left(E_{\mathrm{c,cube}}(28)\right)$: coefficient of variation of elastic modulus of sample, *t* = 28 days.

Material properties (cont.)

Cylindrical strength of sample at *t* days :

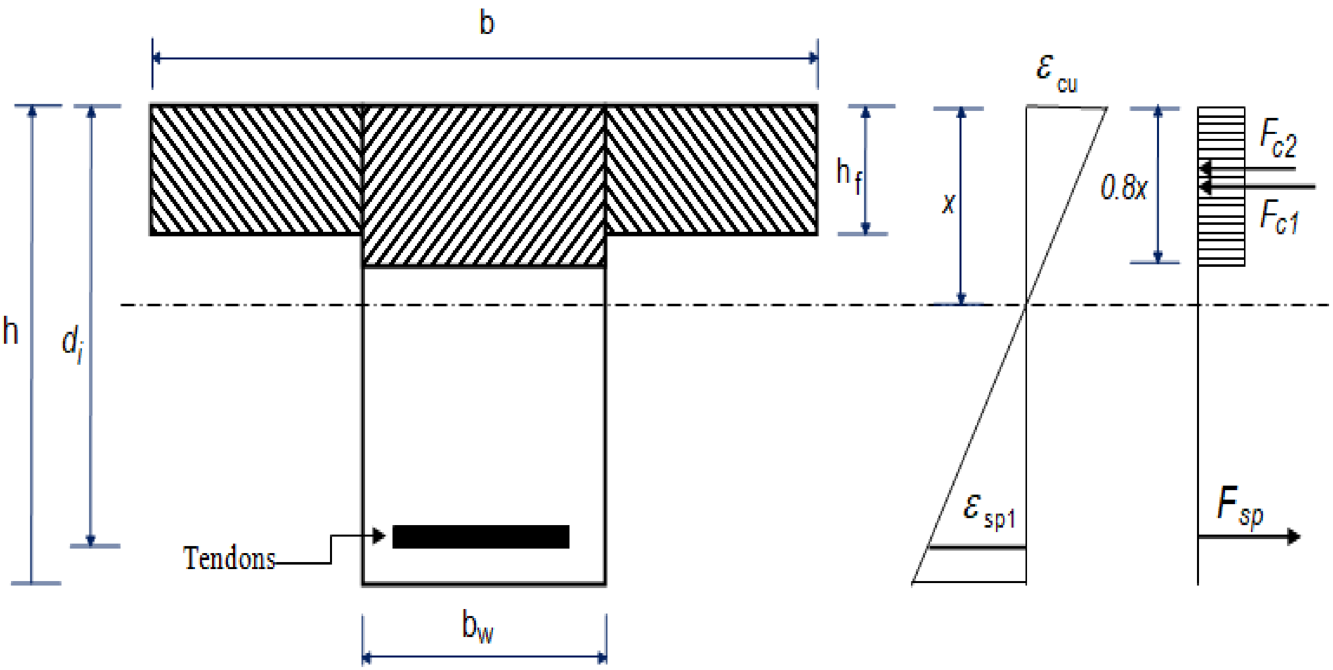
$$f_{c,cyl}(t) = \mu\left(f_{c,cyl}(t)\right) + cov\left(f_{c,cube}(28)\right).\mu\left(f_{c,cyl}(t)\right)$$

$\mu\left(f_{c,cyl}(t)\right)$: mean value of cylindrical strength of sample at age *t*;

$cov\left(f_{c,cube}(28)\right)$: coefficient of variation of cubic strength of sample, t = 28 days.

T-beam bending resistance & deflection

- Computation model:



Height of compression zone is larger than the flange thickness

Ultimate bending moment resistance:

$$M_r = F_{c1}(d - 0.4x) + F_{c2}(d - 0.5h_f)$$

T-beam bending resistance & deflection

F_{c1} : compressive forces (kN) in concrete from web:

$$F_{c1} = 0.8 f_{c,cyl} b_w x,$$

F_{c2} : compressive forces (kN) in concrete from flange:

$$F_{c2} = 0.8 f_{c,cyl} (b - b_w) h_f$$

x : height of compression zone (m):

$$x = \frac{F_{sp} - F_{c2}}{0.8 f_{c,cyl} b_w}$$

T-beam bending resistance & deflection

F_{sp} : total prestressing force (kN) in all layers of tendons after relaxation losses, computed as:

$$F_{sp} = \sum N_{xi} A_p sigma\_pst$$

sigma_pst : prestressing stress (kPa) after relaxation losses:

t (hour)		72	500.000
sigma_pst (kPa)	*sigma_pmax*	*sigma_pmax* × 0.85	*sigma_pmax* × 0.85 × 0.85

sigma_pmax : maximum prestressing stress (kPa) of tendons

T-beam bending resistance & deflection

<u>Height of compression zone is smaller than the flange thickness</u>

Ultimate bending moment resistance: $M_r = F_c(d - 0.4x)$

Compressive force in concrete: $F_c = 0.8\, f_{c,cyl}\, b\, x$

Height of compression zone: $x = \dfrac{F_{sp}}{0.8 f_{c,cyl} b}$

T-beam bending resistance & deflection

Deflection of the T-beam

Instant deflection at middle of the beam in the time when the test of ultimate carrying capacity is conducted:

$$w = w_g + w_p + w_F$$

w_g : deflection (m) due to dead load

w_p : deflection (m) due to prestressing force

w_F : deflection (m) due to induced moment without prestressing.

Probabilistic modelling of T-beam

- Monte Carlo simulation technique:

  Creating 3000 MC samples

  Running the model

  Analyzing the data

Probabilistic modelling of T-beam

- Generation of random variables considering normal distribution:

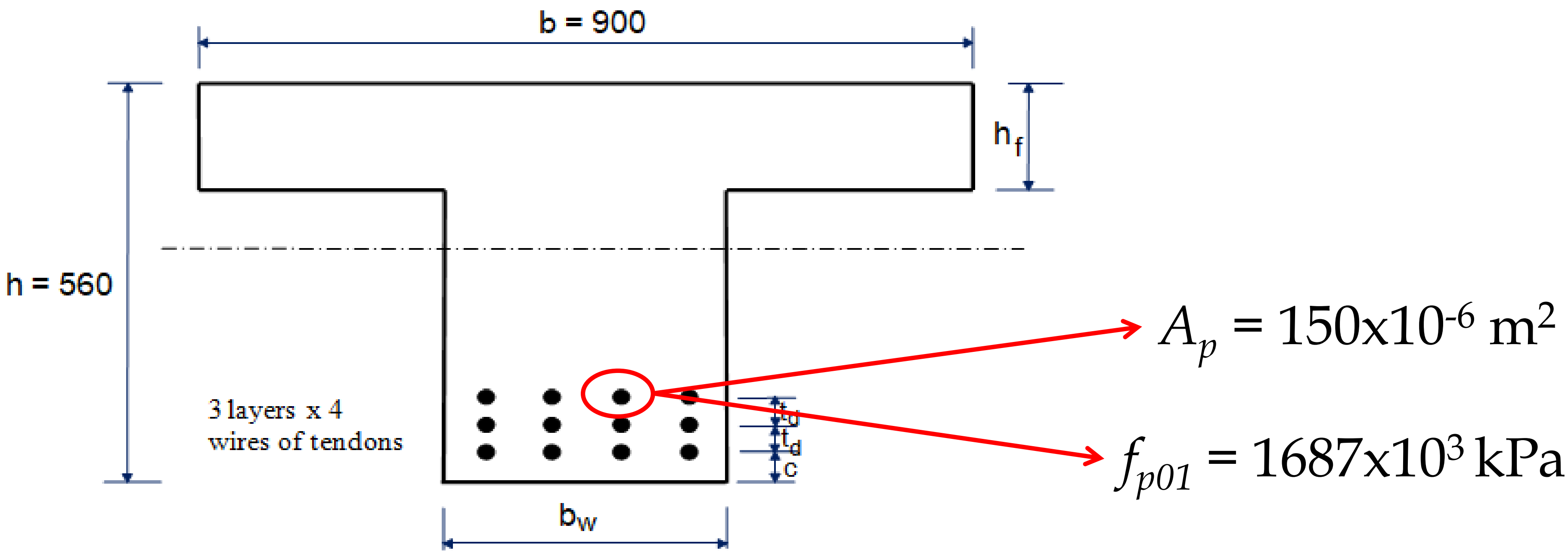
  Transformation formula:

$$N(\mu,\sigma) = \mu + \sigma \times N(0,1)$$

For time dependent parameters:

$$N(\mu,\sigma,t) = \mu(t) + \sigma(t) \times N(0,1)$$

Numerical examples



- effect of conventional reinforcement is neglected
- top layer of tendons was not considered in the calculation

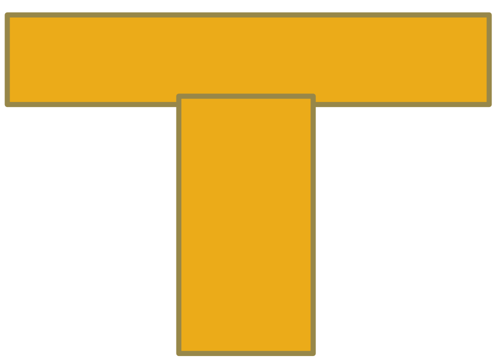
Numerical examples (cont.)

- Deterministic solution:

  Geometry of section A: h = 0.56 m, b = 0.9 m, h_f = 0.18 m, b_w = 0.31 m, c = 0.08 m, t_d = 0.05 m, d = 0.52 m, l = 7 m.

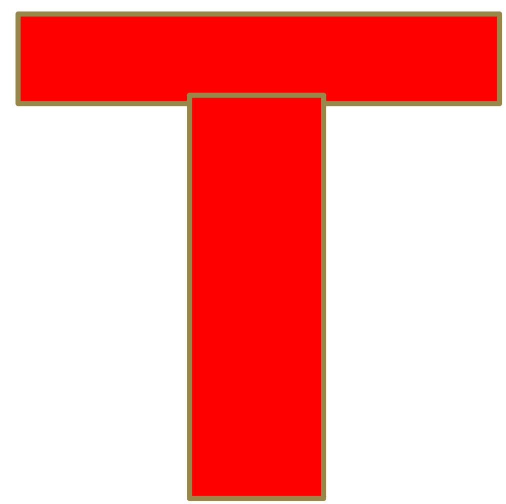
  Geometry of section B: h = 1.2 m, b = 0.9 m, h_f = 0.22 m, b_w = 0.31 m, c = 0.08 m, t_d = 0.05 m, d = 1.12 m, l = 7 m.

A= 0.2798 m^2



Section A

A= 0.5018 m^2



Section B

Numerical examples (cont.)

- Deterministic solution:

Prestressing tendons:

$A_p = 150\times10^{-6}$ m^2,
$f_{p01} = 1687\times10^3$ kPa,
$sigma\_pmax = 1400\times10^3$ kPa.

Limit strain of concrete and steel:

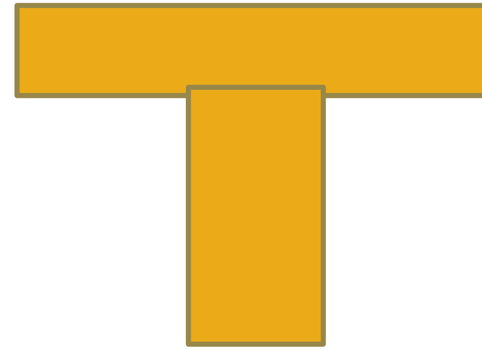
+ $\varepsilon_{cu} = 0.0035$,
+ $\varepsilon_{ud} = 0.02$.

Concrete properties:

+ $\rho = 2390$ kg/m^3,
+ $E_c = 26.522$ kPa,
+ $f_{c,cyl} = 42.531$ kPa.

Numerical example A

- Deterministic solution:

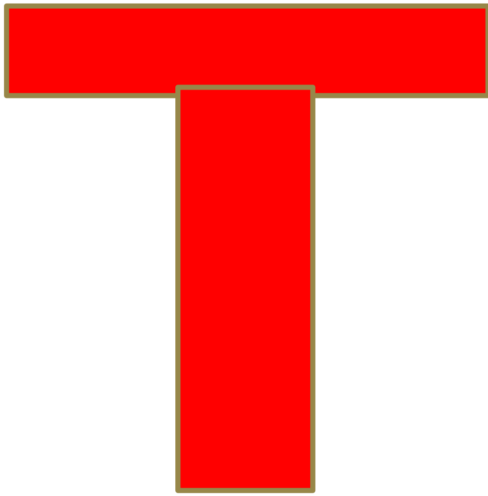


Results from deterministic analysis of considered T-beam (A = 0.2798 m^2)

Deterministic analysis results with A = 0.2798 m^2	t < 3 days = 72 hours (at t = 2 days)	72 hours ≤ t < 500.000 hours (at t = 14 days)	t ≥ 500.000 hours (at t = 28 days)
Bending moment resistance, Mr (kNm)	1,227.4	1,076.1	1,080.5
Vertical deflection, w, at middle of the beam under the ultimate loading (m)	0.0200	0.0135	0.0125

Numerical example B

- Deterministic solution:



Results from deterministic analysis of considered T-beam (A = 0.5018 m^2)

Deterministic analysis results with A = 0.5018 m^2	$t < 3$ days = 72 hours (at $t = 2$ days)	72 hours $\leq t <$ 500.000 hours (at $t = 14$ days)	$t \geq$ 500.000 hours (at $t = 28$ days)
Bending moment resistance, Mr (kNm)	2,739.4	2,361.3	2,365.7
Vertical deflection, w, at middle of the beam under the ultimate loading (m)	0.0047	0.0031	0.0029

Numerical examples (cont.)

- Deterministic solution:

Bending resistance of considered T-section in this study versus bending resistance of rectangular section investigated in (Le *et al.*, 2018)

Age of concrete (days)	Bending moment resistance, *Mr* (kNm)			Increase (+) / Decrease (-) of *Mr*	
	T-section, A = 0.2798m^2 (a)	T-section, A = 0.5018m^2 (b)	Rectangular section, A = 0. 504m^2 (Le *et al.*, 2018) (c)	(a) vs. (c)	(b) vs. (c)
$t = 2$	1,227.4	2,739.4	1,458.4	- 15.8 %	+ 87.8 %
$t = 28$	1,080.5	2,365.7	1,511.8	- 28.5%	+ 56.5%

Numerical examples (cont.)

- Probabilistic solution:

$$E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0.3} E_{cm} \quad \text{(X.1)}$$

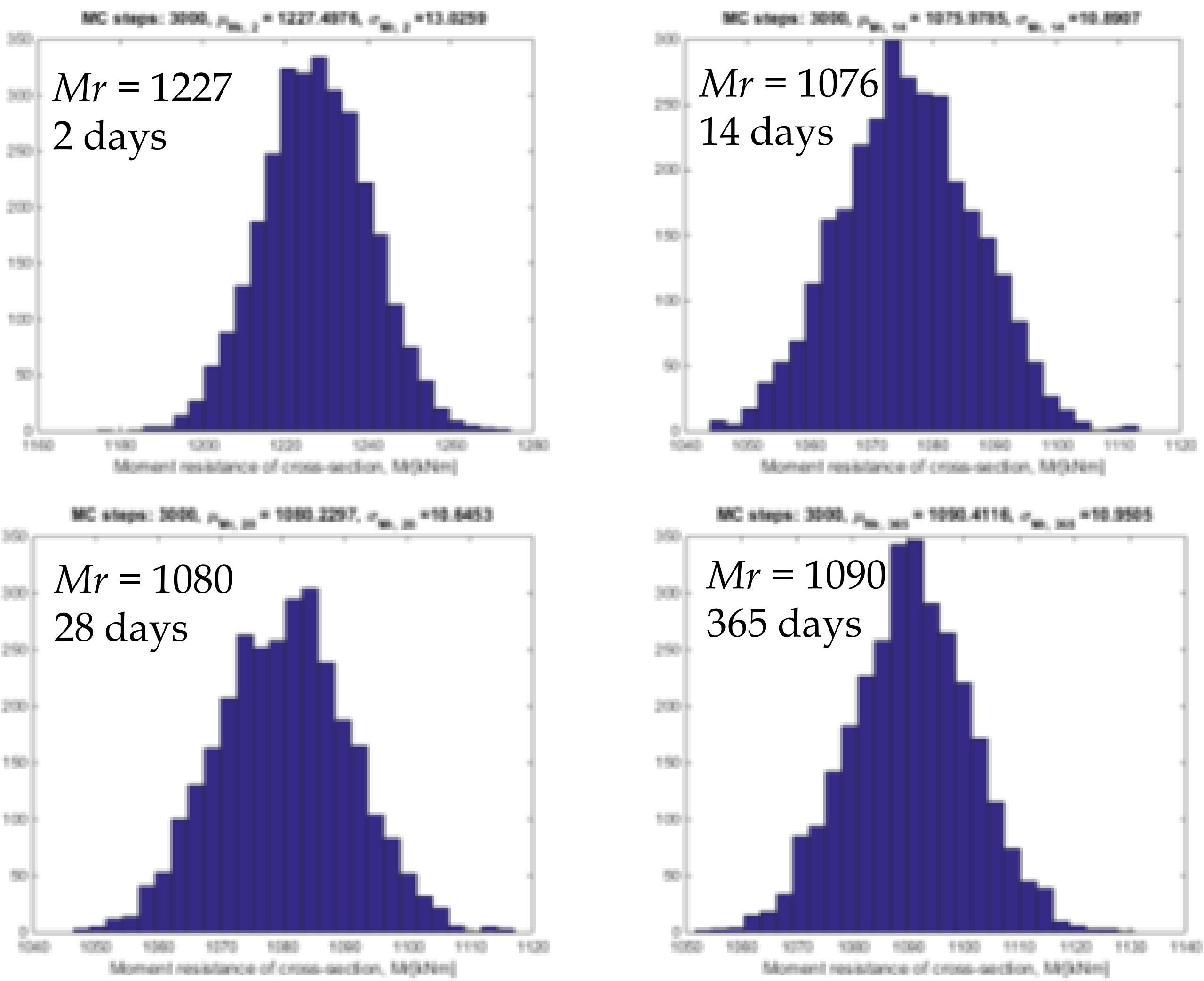
$$f_{cm}(t) = \beta_{cc}(t).f_{cm} \; ; \quad \text{(X.2)}$$

Parameter	Notation	Mean	Coefficient of variation	Transformation
Elastic modulus of concrete (kPa)	$E_{c,cyl}(t)$	eq. X.1	0.0388	$\mu\left(E_{c,cyl}(t)\right)+$ 0.0388×N(0,1)
Concrete strength (kPa)	$f_{c,cyl}(t)$	eq. X.2	0.0388	$\mu\left(f_{c,cyl}(t)\right)+$ 0.0388×N(0,1)(*)
Effective height of the beam (m)	d	0.52	0.0096	d=0.52+ 0.005×N(0,1)
Thickness of the web (m)	b_w	0.31	-	-
Thickness of the flange (m)	h_f	0.18	-	-
Loading span (m)	l	6.85	-	-
Width of flange of the considered beam (m)	b	0.9	-	-
Height of cross section of the beam (m)	h	0.56	-	-
Cross-sectional area of prestressing tendons (m^2)	A_p	$150\text{x}10^{-6}$	-	-
0.1% proof-stress of prestressing steel (kPa)	f_{p01}	1687×10^3	-	-
Thickness of concrete covered layer (m)	c	0.08	-	-

(*): Please be noted that $E_{c,cyl}(t)$ and $f_{c,cyl}(t)$ are considered as uncorrelated parameters here.

Numerical examples (cont.)

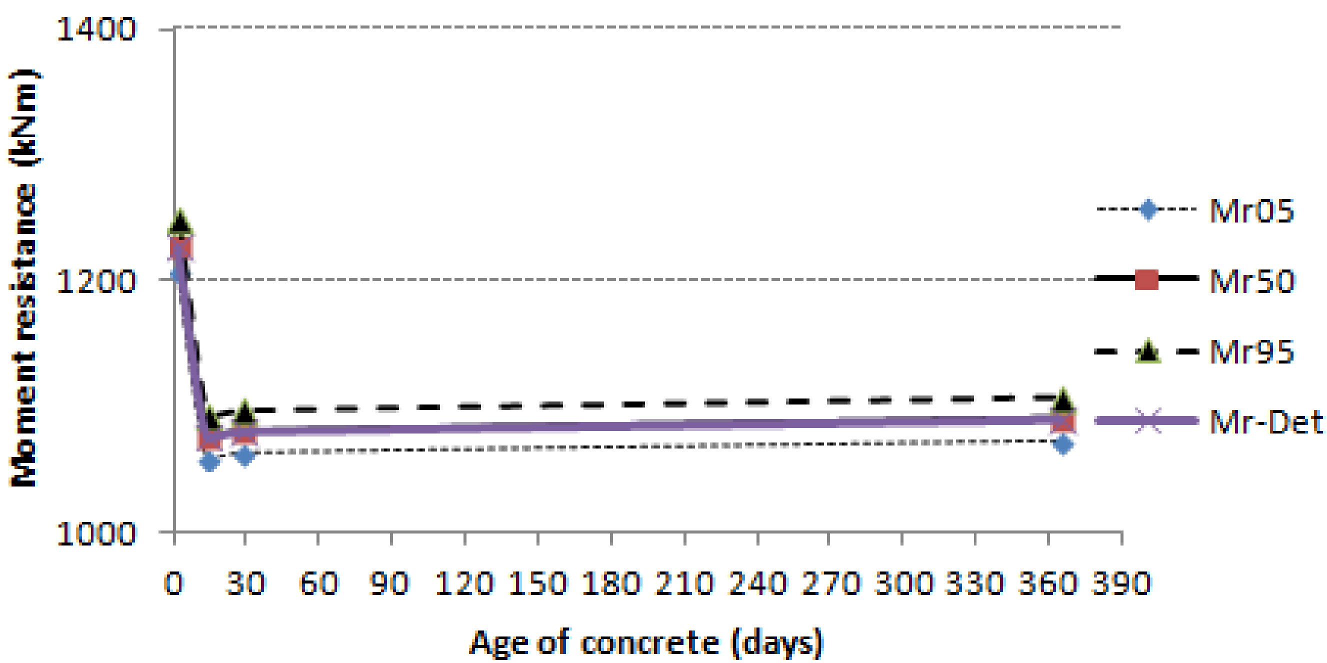
- Distribution of bending moment resistance (kNm):



Numerical examples (cont.)

- Bending moment resistance of cross section of the T-beam

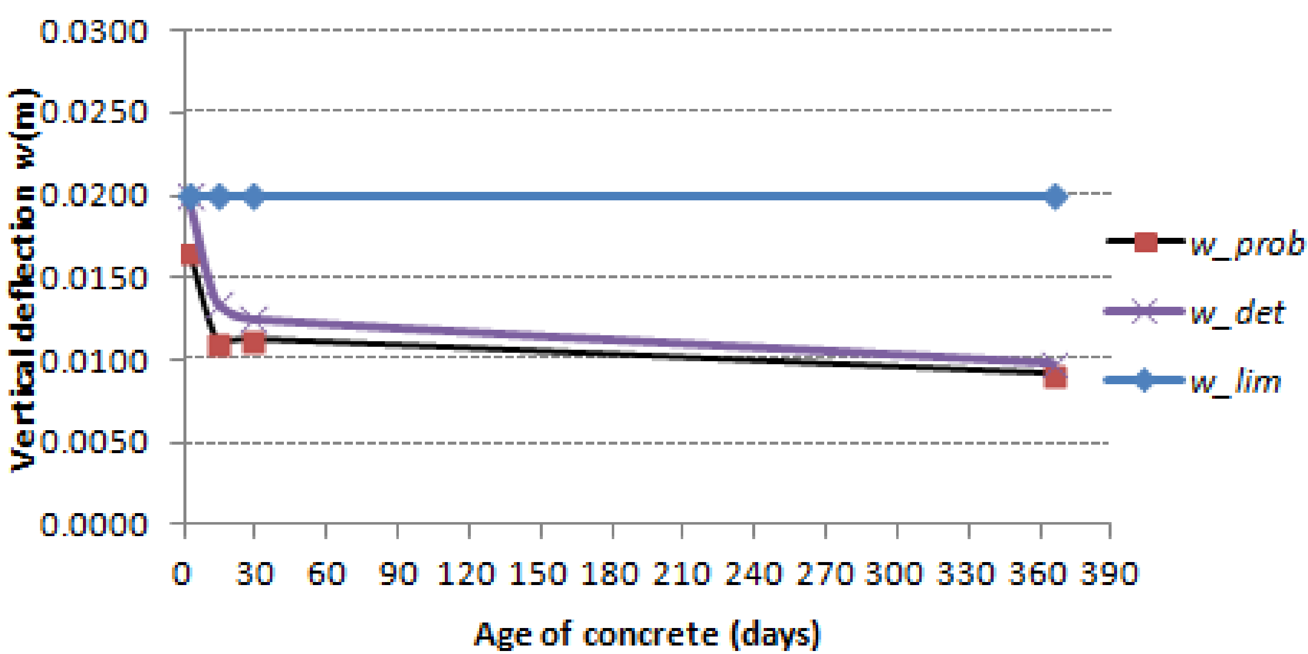
Age of concrete t (days)	Bending moment resistance of the cross section of the T-beam (kNm)			
	M_{r05} (5%)	M_{r50} (50%)	M_{r95} (95%)	Deterministic analysis
2	1,206.072	1,227.498	1,248.923	1,227.4
14	1,058.065	1,075.979	1,093.892	1,076.1
28	1,062.720	1,080.230	1,097.740	1,080.5
365	1,072.400	1,090.412	1,108.424	1,090.5



Numerical examples (cont.)

- Vertical deflection of the T-beam

Age of concrete t (days)	Vertical deflection (m) at the middle of the T-beam	
	Probabilistic solution (maximum value)	Deteministic solution
2	0.0166	0.0200
14	0.0111	0.0135
28	0.0113	0.0125
365	0.0092	0.0098



Sample section optimization

- Input parameters for the optimization

Geometry of the section:

h = 0.56 m, b = 0.9 m, c = 0.08 m,
t_d = 0.05 m, d = 0.52 m, l = 7 m.
h_f and b_w are variables

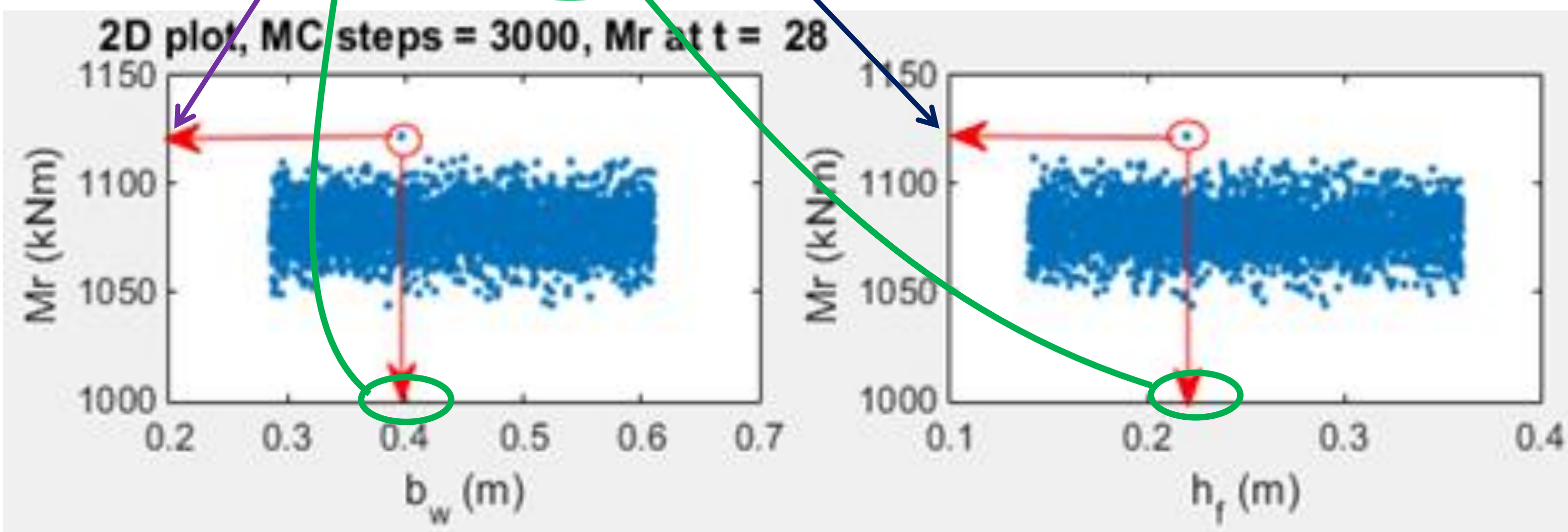
Distributions:

$E_{c,cyl}(t)$: normal distribution
$f_{c,cyl}(t)$: normal distribution
h_f : uniform distribution
b_w : uniform distribution

Sample section optimization (cont.)

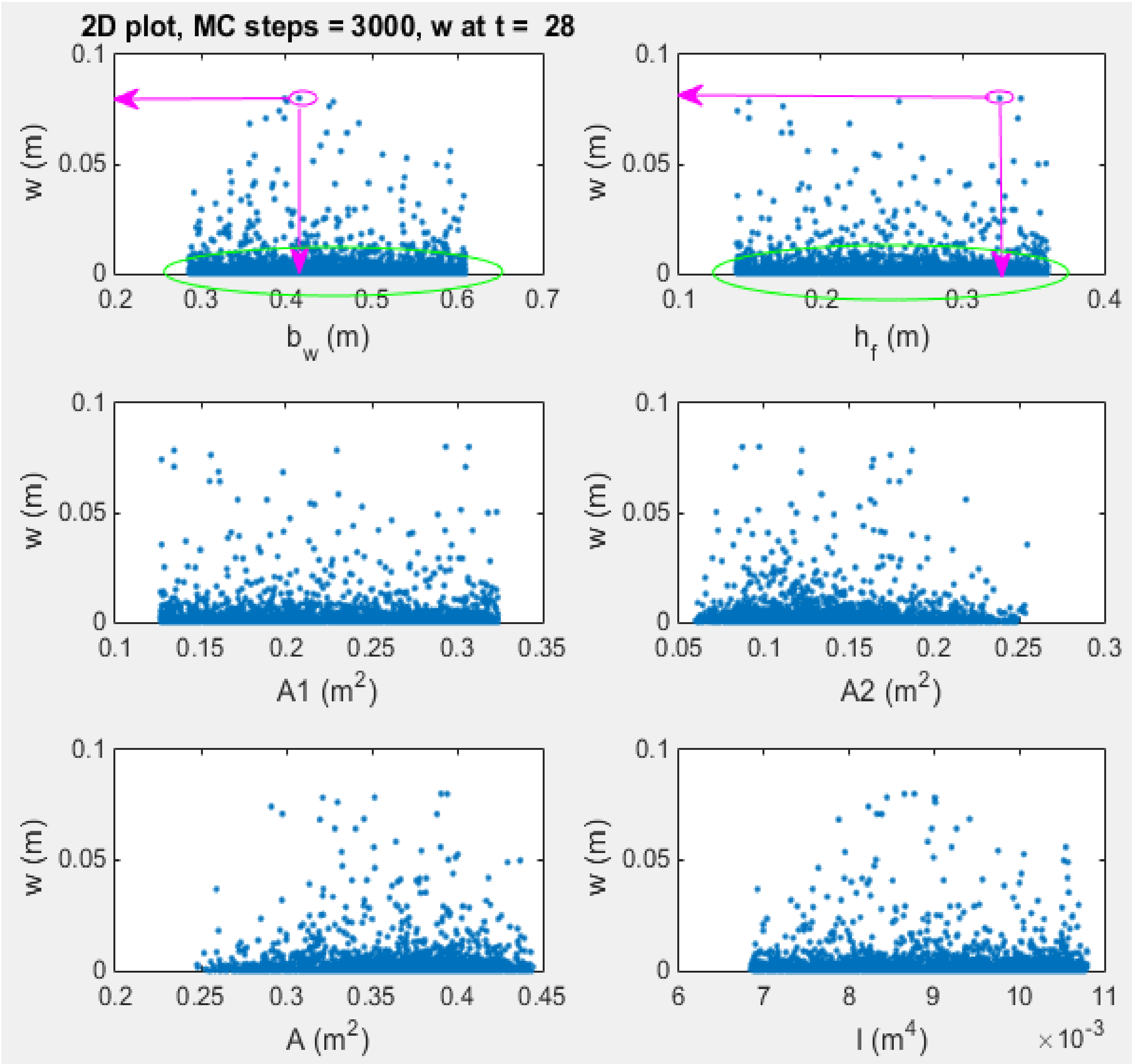
- Partial results in case of t = 28 days

Age of concrete, t (days)	Maximum resistance, max_Mr = 1122.3 kNm		Minimum vertical deflection, $min\_w = 6.641 \cdot 10^{-8}$ m		Maximum vertical deflection, $max\_w = 0.080$ m	
	b_w (m)	h_f (m)	b_w (m)	h_f (m)	b_w (m)	h_f (m)
28	0.396	0.220	0.501	0.358	0.415	0.326



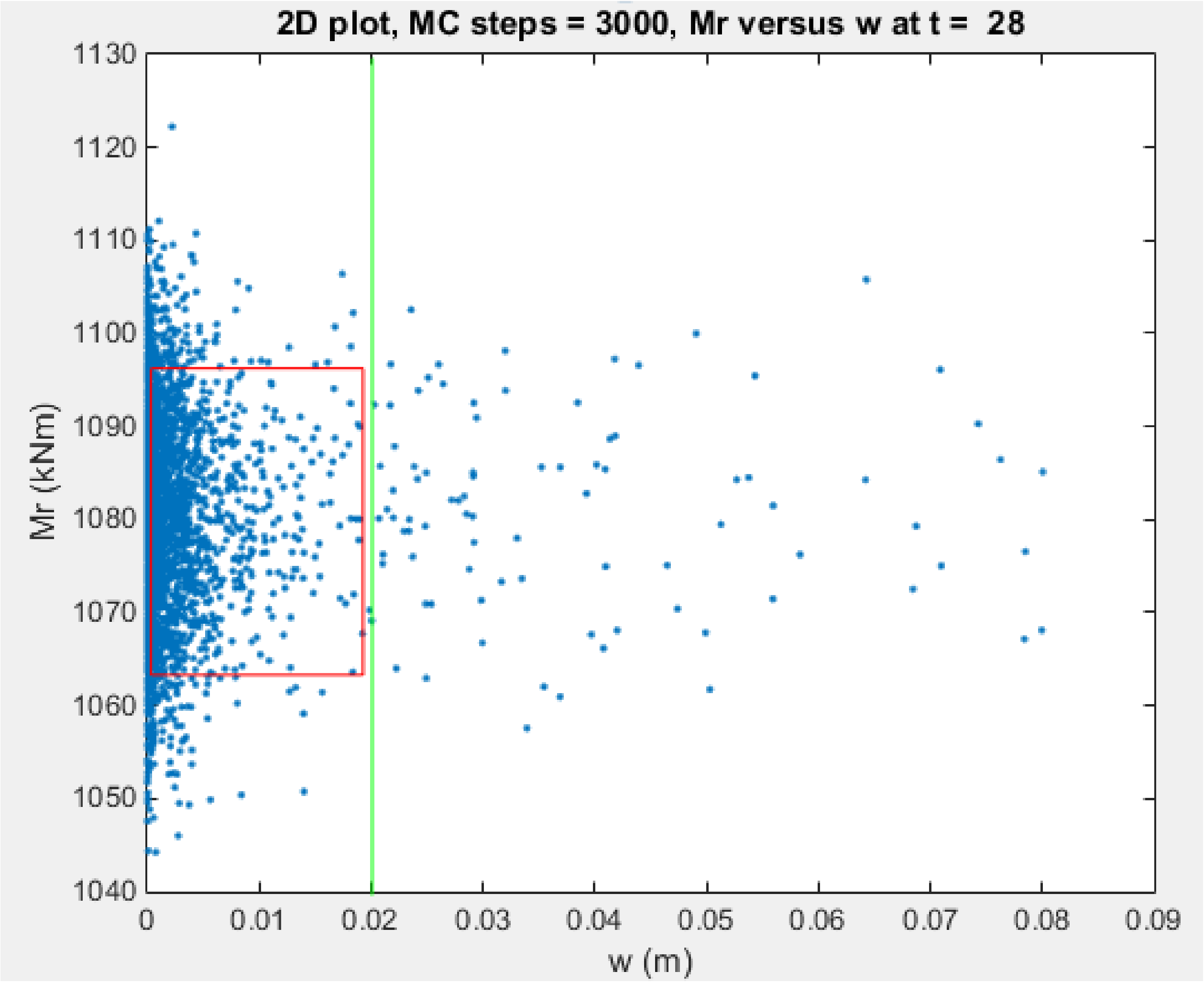
Sample section optimization (cont.)

- Partial results in case of t = 28 days



Sample section optimization (cont.)

- Partial results in case of t = 28 days



Sample section optimization (cont.)

- Overall results

Two conditions of section optimization:

✓ vertical deflection (*w*) of the cross section is always smaller than maximum allowable vertical deflection (*w_lim*);

✓ bending moment resistance of the cross-sectional T-beam (M_r) is larger than minimum value of bending moment resistance of the beam at concrete age of 28 days.

Sample section optimization (cont.)

- Overall results

Option No.	Vertical deflection			Bending resistance			Area of cross section			Score (%)
	w (m)	Percentage of the best (%)	weight	M_r (kNm)	Percentage of the best (%)	weight	A (m^2)	Percentage of the best (%)	weight	
1	$\boxed{7.6\times10^{-6}}$	93	0.3	$\boxed{1098.8}$	95	0.2	$\boxed{0.2743}$	97	0.5	95.4
2	1.3×10^{-6}	97	0.3	1100.9	96	0.2	0.2897	94	0.5	95.3
3	1.2×10^{-7}	99	0.3	1101.5	97	0.2	0.3016	90	0.5	94.1
4	7.8×10^{-7}	98	0.3	1095.3	90	0.2	0.2940	93	0.5	93.9
5	2.9×10^{-6}	96	0.3	1090.6	81	0.2	0.2877	95	0.5	92.5
6	8.1×10^{-6}	93	0.3	1086.9	71	0.2	0.2593	99	0.5	91.6
7	8.6×10^{-6}	93	0.3	1092.6	85	0.2	0.2933	93	0.5	91.4
8	6.4×10^{-5}	81	0.3	1092.8	86	0.2	0.2532	99	0.5	91.0
9	6.3×10^{-5}	81	0.3	1100.1	96	0.2	0.2810	95	0.5	91.0
10	1.1×10^{-6}	97	0.3	1098.4	94	0.2	0.3110	86	0.5	90.9

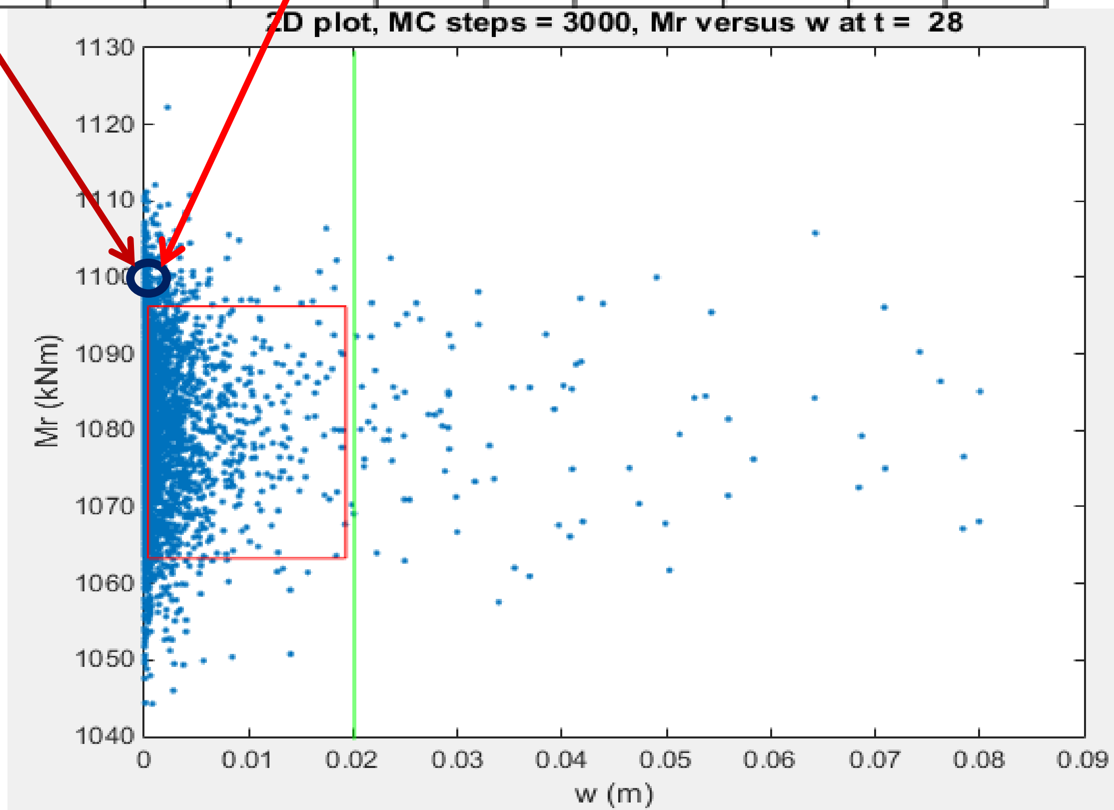
$$\left[\begin{array}{l} b_w = 0.3079\ \text{m}, \\ h_f = 0.1721\ \text{m}. \end{array}\right.$$

Sample section optimization (cont.)

- Overall results

Option No.	Vertical deflection: w (m)	Vertical deflection: Percentage of the best (%)	Vertical deflection: weight	Bending resistance: M_r (kNm)	Bending resistance: Percentage of the best (%)	Bending resistance: weight	Area of cross section: A (m²)	Area of cross section: Percentage of the best (%)	Area of cross section: weight	Score (%)
1	7.6×10^{-6}	93	0.3	1098.8	95	0.2	0.2743	97	0.5	95.4

$$\begin{cases} b_w = 0.3079 \text{ m} \\ h_f = 0.1721 \text{ m} \end{cases}$$



Summary

A simple probability-based model for the bending resistance computation of a simply supported pre-stressed concrete T- beam is set up.

Numerical codes for evaluation of bending resistance and deflection of the T-beam were composed.

Simple MC-based section optimization of the T-beam.

Conclusions

- Computation of bending resistance and vertical deflection over the time of a prestressed precast concrete T-beam was done.

- Randomness of selected parameters were considered.

- The performance of the T-beam under prestressing was studied via numerical example by application of Monte Carlo simulation technique.

Conclusions (cont.)

➢ Dimensions of T-section of the beam in the example were also optimized based on:

- maximum bending resistance
- best performance of vertical deflections.

➢ Optimization results are consistent with those of bending resistance computation part.

The built procedure for section optimization seems to be applicable for prestressed concrete T-beam.

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Conclusions (cont.)

This work should be further developed with:

- taking into account of tendons in the top part of the section and longitudinal reinforcement;
- considering impacts of shear resistance;
- studying effects of cracking of the beam.

Thank you for your attention