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4. A correction of fatigue characteristics of concrete with respect to age of specimens

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4.1. Introduction

Many structures are often subjected to repetitive cyclic loads of high stress amplitude. Examples of such cyclic loads include automobile and train traffic, machine vibration and wind action. The phenomenon known as material fatigue, a process in which progressive and permanent internal damage occurs in materials subjected to repeated loading, is a serious problem also for concrete structures such as bridges, tunnels, airport/highway pavements, railway sleepers, etc. Concrete is a highly heterogeneous material and the processes occurring within its structure and leading to its degradation under cyclic loading are more complicated in comparison to those affecting metals (see Lee & Barr 2004). This is one reason why the understanding of fatigue failure in cementitious composites is still lacking in comparison to that of ferrous materials, even though concrete is a widely used construction material.

Fatigue tests of concrete materials and structures are expensive, and for this reason numerical modelling (Pryl et al. 2010) can represent a powerful approach for the prediction of the damage process and fatigue life of such materials under different service conditions. For the effective and correct use of a numerical (material) model it is often necessary to tune its parameters using data obtained during experiments. The correct evaluation of such data is becoming a prerequisite for the correct use of numerical models in practice.

Therefore, the topic of this chapter is the issue of determining fatigue characteristics of selected cement-based composites. In order to achieve relevant fatigue test evaluation which takes into account the age of the specimens, a correction procedure of the measured data based on static compressive strength measurements covering the time interval of the fatigue tests was suggested and verified. The procedure is applied to Wöhler curves obtained from cyclic three-point bending tests of beam specimens with a central edge notch.

4.2. Theoretical background

In practice, the structures are subjected to random fatigue loading. This process of loading is very difficult to model in the laboratory conditions, therefore there is an effort to simplify course of loading. The most frequent case is the replacement of the random loading by sinusoidal load, which were also used in experiments described later.

As in the case of static tests, different loading arrangements have been used in fatigue testing, including compression, tension and bending tests. The most common method of fatigue testing, by far, is via flexural tests (Lee & Barr 2004).

The fatigue loading is usually divided into two categories (RILEM 1984) i.e. low-cycle and high-cycle loading. Low-cycle loading involves the application of a few load cycles at high stress levels. On the other hand, high cyclic loading is characterized by a large number of cycles at lower stress levels. In this chapter, the attention will be paid on high-cycle fatigue.

Various approaches have been used to assess the fatigue life of structural members in recent years. The generally accepted approach in engineering practice is based on empirically derived *S–N* diagrams known as Wöhler curves (stress *S* vs. the number of cycles to failure *N*), see Figure 4.1. Fatigue tests are characterized by relatively large scattering of measured lifetime values at individual stress levels, therefore for an approximate determination of the Wöhler curve, several specimens have to be tested at each stress level. A minimum of 8–12 specimens of tested material is necessary to determine Wöhler curve. For a more accurate determination of its course, or for its statistical evaluation, 15–20 specimens is needed.

S–N test data are usually displayed in logarithmic co-ordinates, where its course is approximated by oblique and horizontal line. The oblique line approximates the set of completed tests. The horizontal line, which represents the fatigue limit, passes through a set of unfinished tests (run outs) in such a way that it corresponds to the highest stress level of only unfinished tests.

Unlike ferrous metals, concrete does not appear to have a fatigue limit. It has been reported that plain concrete subjected to repeated uniaxial tensile stresses exhibits no fatigue limit under 2×10^6 cycles (Satio & Imai 1983). Hence there is no known stress level below which the fatigue life of plain concrete will be infinite (Lee & Barr 2004).

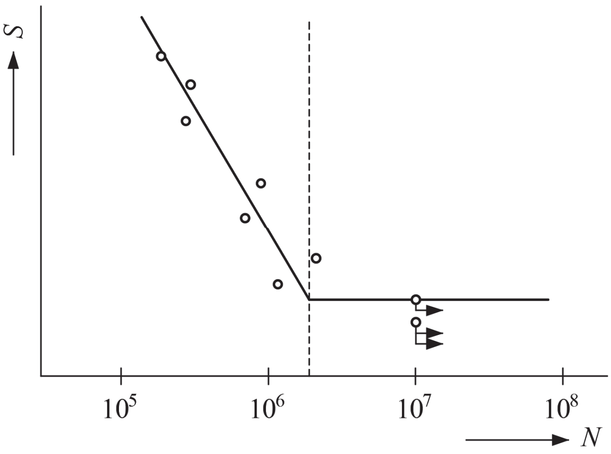

Fig. 4.1. Typical *S–N* (Wöhler) curve (according Věchet & Král 2007).

There are several mathematical descriptions of Wöhler curve:

Basquin function (Weibull 1961):

$$S = a \cdot N^b \tag{4.1}$$

Where:

S	–	applied stress,
N	–	number of cycles,
a	–	material parameter,
b	–	material parameter.

Kohout–Věchet model (Kohout &Věchet 2001):

$$log\frac{\Delta\sigma}{\Delta\sigma_\infty} = b \cdot log\left(\frac{N+B}{N+C}\right) \tag{4.2}$$

Where:

$\Delta\sigma$	–	range of fatigue stress (variables),
$\Delta\sigma_\infty$	–	permanent fatigue limit (limit value of fatigue stress for infinite number of cycles),
N	–	number of cycles (variable),
B	–	material parameter,
C	–	material parameter,
b	–	material parameter.

Another possibility is to use the statistical evaluation of dynamic tests, e. g. the Weibull non-linear regression model developed by Castillo (Castillo & Fernández-Canteli 2009):

$$S = exp\left(\frac{\left(log\left(\frac{1}{1-P}\right)\right)^{1/\beta} \cdot \delta + \lambda}{\log N - B} + C\right) \tag{4.3}$$

Where:

S	–	fatigue stress,
N	–	number of cycles (variable),
P	–	probability of failure,
B	–	model parameter, threshold value for N or the limit number of cycles,
C	–	model parameter, threshold value for S or the endurance limit,
β, δ	–	model parameter, shape and scale parameters of the Weibull distribution,
λ	–	model parameter, parameter fixing the position of the zero probability curve.

4.4. Materials and methods

An extensive laboratory experiment was conducted on a set of specimens of plain C30/37 and C45/55 class concrete. The specimens were used to determine the values of fundamental fracture characteristics and related fatigue parameters using static fracture and dynamic experiments. The partial result of this extensive experiments were published in following publication: Seitl et al. 2012, Šimonová et al. 2013, Šimonová et al. 2014.

As was already mentioned, the fatigue experiments lasted for a long time, which is problematic from the point of view of the ageing of the specimen material. In this case, the fatigue tests were performed between 28 and 160 days of specimen's age. Therefore, the correction procedure of the measured data based on static compressive strength measurements covering the time interval of the fatigue tests was suggested and verified (Šimonová 2013).

The procedure is applied to Wöhler curves (eq. 4.1) obtained from cyclic three-point bending tests of beam specimens with a central edge notch (see Figure 4.2). The nominal dimensions of the beams were 100×100×400 mm; span length 300 mm. The initial notches were made by a diamond bladed saw. Note that the depth of the notches was 10 mm (see Figure 4.3). The fatigue experiments were carried out in a computer-controlled servo hydraulic testing machine (INOVA–U2). The controlled values for temperature and relative humidity were 22±2 °C and 50%.

Fatigue testing was conducted under load control. The stress ratio $R = P_{min}/P_{max} = 0.1$, where P_{min} and P_{max} refer to the minimum and maximum load of a sinusoidal wave in each cycle. The load frequency used for all repeated-load tests was 10 Hz. The number of cycles before failure was recorded for each specimen. Concrete specimens were loaded in the range of high-cycle fatigue; therefore, the upper limit to the number of cycles to be applied was selected as 2 million cycles. The test finished when the failure of the specimen occurred or the upper limit of loading cycles was reached, whichever occurred first.

Fig. 4.2. Three-point bending fatigue test configuration.

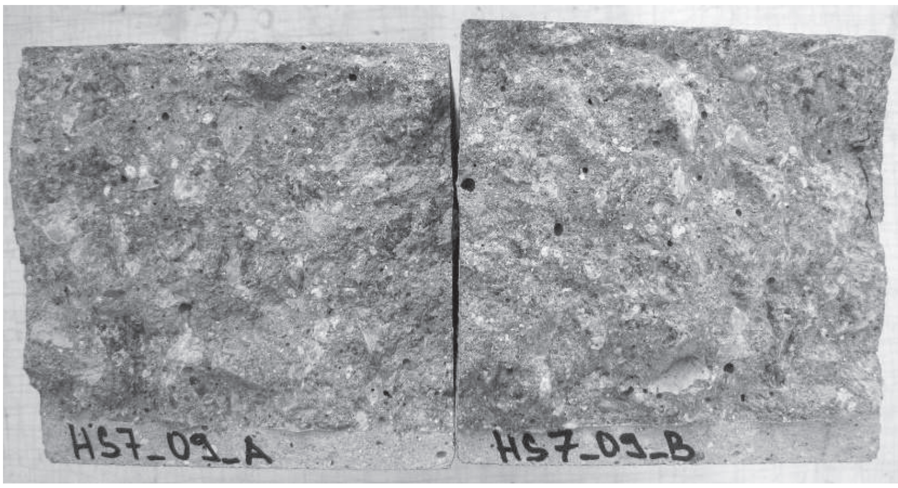

Fig. 4.3. Illustration of selected tested specimen after three-point bending fatigue test.

4.4. Advanced approximation of compressive strength values of concrete specimens

The suggested correction procedure allows more accurate determination of the fatigue parameters corresponding for the age of the specimens when the dynamic test was performed.

At first, analytical expressions were determined for regression curves as approximations of the selected mechanical fracture parameter values over time – compressive strength, modulus of elasticity, effective fracture toughness and specific fracture energy in the following way. In the first instance the relative values of individual parameters for all examined ages of specimens were obtained: the values of parameters obtained from measurement were divided by the appropriate mean average value for specimens at the age of 28 days. In the next step adaptation data for individual parameters were plotted in charts depending on the age of specimens. The analytical expressions for simple approximation (regression) curves were determined. Power, logarithmic and polynomial functions were used (EXCEL software). The coefficient of determination (R^2) was obtained for each type of regression curve.

From the analytical equations determined for the simple approximation curves of all mentioned mechanical fracture parameter values of plain C30/37 and C45/55 class concrete specimens follows that the closest approximations, based on the coefficient of determination value, were achieved for compressive cube strength values (Šimonová et al. 2011, 2013). For this reason, an advanced approximation curve was used for the compressive strength values, which more accurately describes the development of the compressive strength over time and not only in the interval in which dynamic tests were performed, as was in the case for simple approximation curves.

The cube specimens with edge lengths of 150 mm were used for determination of the compressive strength values. Because of the gradually increasing age of the concrete specimens during the dynamic tests, the specimens were tested at the age of 28, 98 (91 for C45/55 class concrete) and 159 days to cover whole interval of performing of dynamic tests. A summary of the compressive cube strength results for C30/37 and C45/55 class concrete is given in Table 4.1 and 4.2.

Table. 4.1. The compressive cube strength values [MPa] of C30/37 class concrete specimens at investigated ages.

Speci-men	f_{c28}				Speci-men	f_{c98}	Speci-men	f_{c159}
	Value	Mean value	Standard deviation	COV [%]		Value		Value
HS2_1	58.23	57.2	1.11	1.9	HS2_4	69.02	HS2_7	74.36
HS2_2	57.31				HS2_5	70.74	HS2_8	72.30
HS2_3	56.02				HS2_6	71.01	HS2_9	69.74

Table. 4.2. The compressive cube strength values [MPa] of C45/55 class concrete specimens at investigated ages.

Speci-men	f_{c28}				Speci-men	f_{c91}	Speci-men	f_{c159}
	Value	Mean value	Standard deviation	COV [%]		Value		Value
HS6_1	77.01	76.1	1.65	2.2	HS6_4	80.35	HS6_7	92.54
HS6_2	74.23				HS6_5	84.57	HS6_8	92.58
HS6_3	77.17				HS6_6	81.82	HS6_9	88.50

As in the case of simple approximation curves, the values of compressive strength obtained from measurement were divided by the appropriate mean value for specimens at the age of 28 days, thereby the relative values of compressive strength were determined. In the next step the relative values were then approximated by the function according to Abdel-Jawad 2006 in the modified form:

$$\frac{f_c(t)}{f_{c28}} = a \cdot \left(1 - e^{-b(t)^c}\right) \tag{4.4}$$

Where:

$f_c(t)$ – compressive strength at age t days,

f_{c28} – compressive strength at the age of 28 days,

a – $a = f_{c\infty}/f_{c28}$, coefficient represents an asymptote to an approximation curve expressed as the ratio between the theoretical compressive strength at the age $t = \infty$ and the determined mean value of compressive strength at the age of 28 days,

b, c – the exponential part of the equation with coefficients b and c expresses the degree of the time-dependent change in compressive strength at the interval $t = (0; \infty)$, which is generally dependent on the parameters of used concrete mixture and also on the environment conditions in which the specimens are stored.

The approximation was performed with the method of least squares using the genetic algorithms implemented in the open source Java package GA – Implementation of simple genetic algorithm (Frantík 2011).

Figure 4.4 shows the course of the approximation curve (4.4) for both classes of concrete with resulting coefficients *a*, *b* and *c*; The *x* symbol in the equation denotes the time in days, *y* the dimensionless relative value of the compressive cube strength, and R^2 is the dimensionless coefficient of determination.

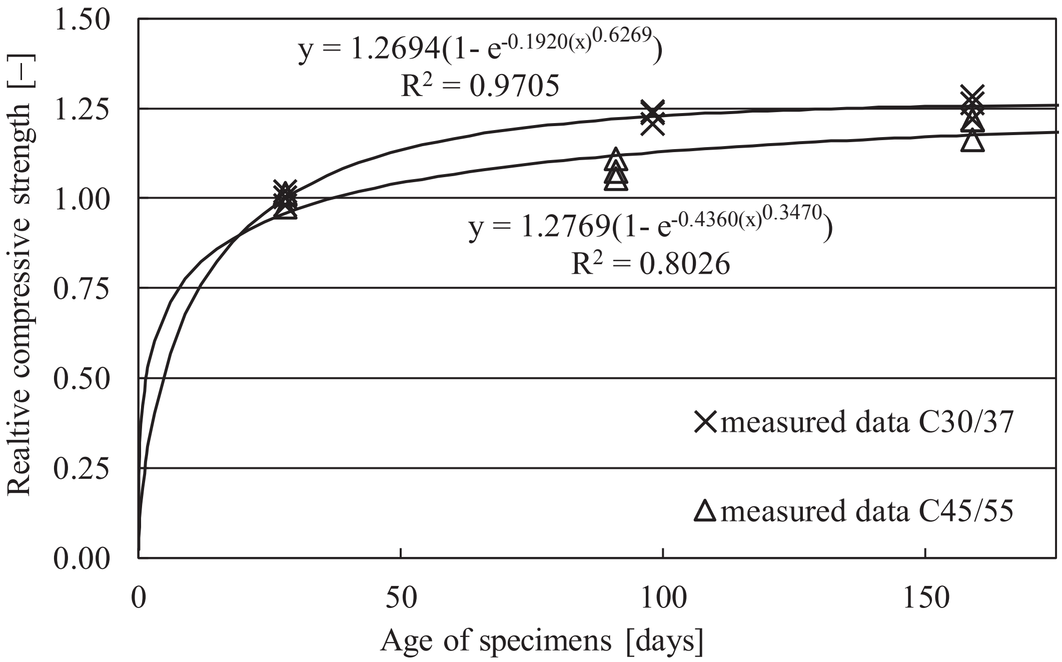

Fig. 4.4. Approximation curves for relative compressive cube strength.

4.5. Results of the fatigue fracture tests

The results of the fatigue tests under a varying maximum bending stress level are summarized in Figs. 4.5 and 4.6 for C30/37 and C45/55 class concrete, respectively, where the maximum bending stress (*S*) applied during the fatigue experiments is plotted against the logarithm of the number of cycles to failure (*N*).

In an ideal, theoretical case, all specimens at a certain stress level would fail after the same number of cycles. However, the fatigue behaviour of a heterogeneous material like concrete is far from being ideal, so the results are usually highly scattered. Accordingly, it is necessary to determine not only the analytical expression of the relevant *S–N* curve but also a measure of the scatter, such as the coefficient of determination R^2.

According to (4.1), the power function and the coefficient of determination for C30/37 class concrete are as follows:

$$S = 4.7256 \cdot N^{-0.0230} \text{ and } R^2 = 0.3806 \tag{4.5}$$

and for C45/55 class concrete:

$$S = 5.9227 \cdot N^{-0.0345} \text{ and } R^2 = 0.7289. \tag{4.6}$$

The coefficient of determination of *S–N* curve determined directly from the measured values is relatively low for both strength classes of concrete. In order to obtain correct values of fatigue characteristics corresponding to specimen age at which the fatigue tests were performed, the measured data were divided by the coefficients determined from the above introduced approximation curves of the compressive strength in time. The measured data was standardized to a specimen age of 28 days using this procedure.

For illustration, in Figs. 4.5 and 4.6, in addition to the measured data, the corrected data using the coefficients obtained from the advanced approximation curve of the relative compressive strength values (see Fig. 4.4) are also plotted, including the analytical expression of *S–N* curve and coefficient of determination. In case of C30/37 class concrete, this procedure led to a significant increase of the value of the dimensionless coefficient of determination from 0.38 to 0.70. In case of C45/55 class concrete an increase of the coefficient of determination was also achieved, although not in such extent, from 0.73 to 0.88. The obtained results can be considered as evidence of the efficacy of the described correction procedure.

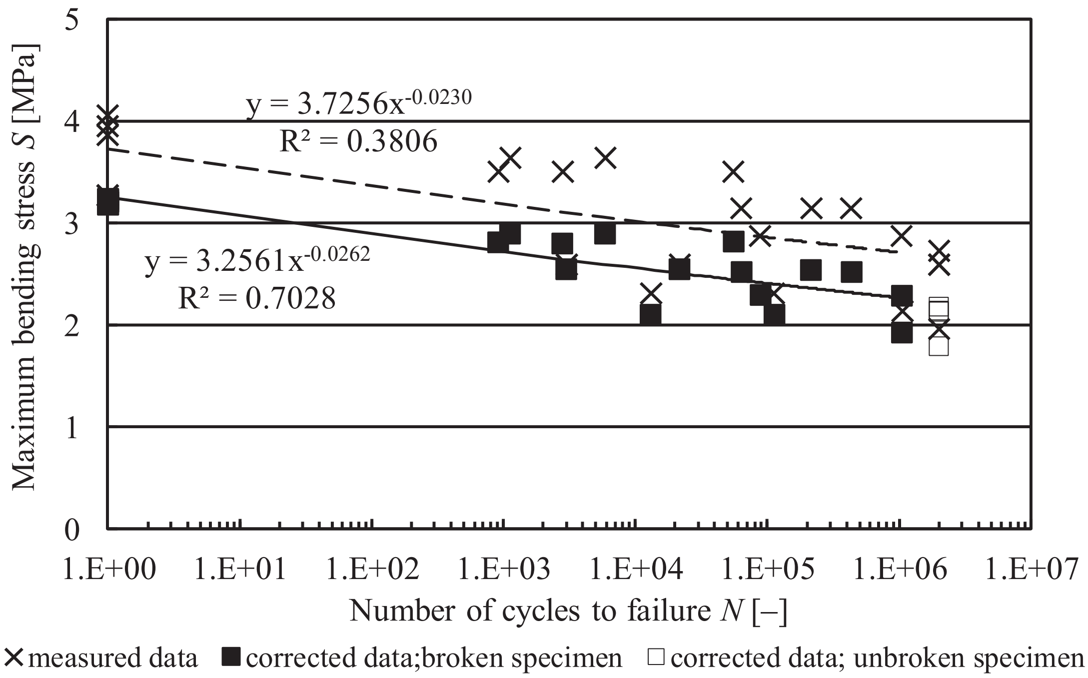

Fig. 4.5. *S–N* curves from the C30/37 class concrete.

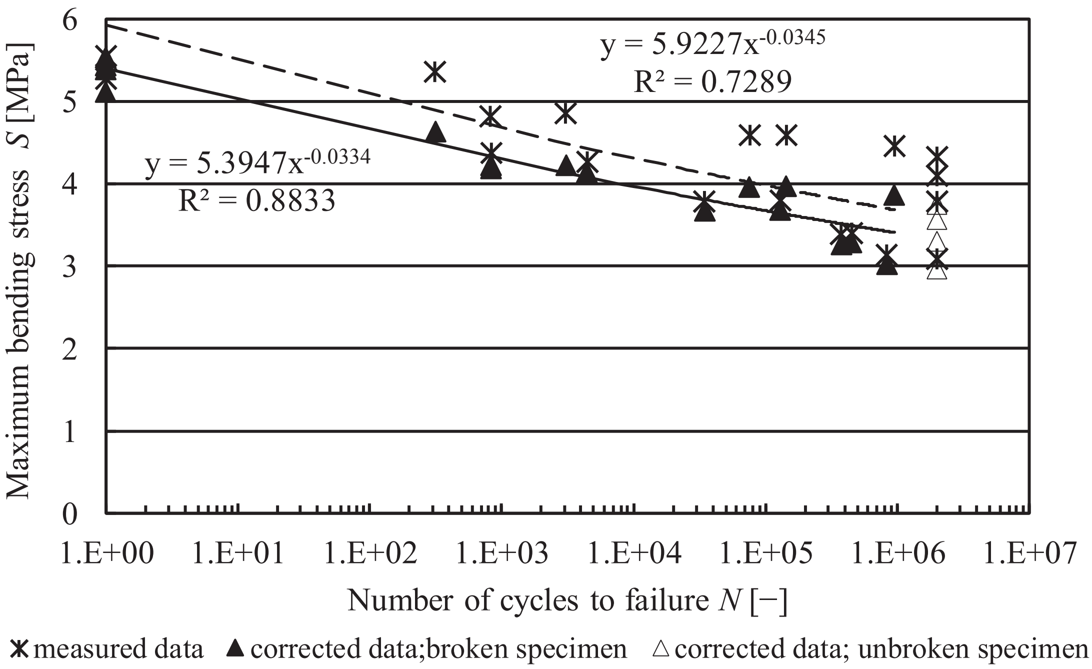

Fig. 4.6. *S–N* curves from the C45/55 class concrete.

4.6. Conclusions

A rather complex procedure of correction of the values of the basic fatigue parameters – Wöhler curves – of C30/37 class concrete specimens with initial central edge notch tested in three-point bending cyclic fracture tests was introduced in this chapter. The correction procedure is based on approximation of static compressive strength measured data covering the time interval of the fatigue tests. The experiment was repeated with the same extent at C45/55 strength class concrete specimens. It can be stated that correction for both sets of concrete specimens led to an increase in the coefficient of determination of Wöhler curve, which can be considered as proof of its effectiveness. The introduced correction procedure was used also for another sets of concrete specimens with an alkali-activated based binder (see Seitl et al. 2014, 2016) and also in this case the increase of coefficient of determination of Wöhler curve was achieved.

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