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8. Simulation quality of the probability of the reinforced concrete corrosion initiation evaluation

Tuan Le Duc[1,2], Petr Konečný[2]

[1] *Faculty of Civil Engineering, Saigon Technology University, 180 Cao Lo Str., Ward 4, Dist. 8, HCMC 700000, Vietnam*

[2] *VSB – Technical University of Ostrava, Faculty of Civil Engineering, Department of Structural Mechanics, L. Podeste 1875, 708 33 Ostrava-Poruba, Czech Republic*

8.1. Introduction

Steel reinforced concrete (RC) is widely used in building structures while it is rather vulnerable to harsh environment such as chloride exposure. Therefore, durability of RC structures during their service lifetime needs to be controlled to ensure proper function and performance. As a result, special attention should be paid during the durability design of RC structures in chloride environment. Many researchers focussed on the development of durability design of RC structures in recent decades (see e.g. Collepardi et al., 1972, Stewart & Rosowsky, 1998, Weyers et al., 1998, Hooton et al., 2001, Marsavina et al., 2009, Szweda&Zybura, 2013, Bentz et al. 2013, Tang S.W. et al., 2015). Since the concrete is heterogeneous material and also the other parameters governing the durability of reinforced concrete structures shows high variability, the probabilistic approach gained wider support in recent years (Keršner et al., 1996, Tikalsky, et al., 2005, Konečný, et al., 2007, Vořechovská et al., 2009, Konečný et al., 2011, Teplý & Vořechovská, 2012, Ghosh et al., 2017). Thus, probability-based design method has been a clear trend due to its ability to capture the effects of random interaction of interrelated phenomena in comparison with deterministic method.

The Monte Carlo type technique is often applied for the evaluation of probability of limit state exceedance. Such analysis with simple Monte Carlo simulation might require very large computational effort in comparison with more efficient procedures such as stratified simulation techniques and advanced simulation techniques (McKay et al., 1979, Melchers, 1989, Novák et al., 1998). The efficiency of these methods has been proved in case of low probability events. However, the issue of question herein is efficiency of the enhanced procedures in the evaluation of probability of time-dependent events such as durability related to corrosion initiation. To answer this question, a comparative assessment of durability analysis of illustrative ideal RC bridge deck without cracks with three different simulation techniques of Monte Carlo type is carried out: the simple simulation Monte Carlo, a stratified simulation technique, Latin Hypercube Sampling (McKay et al., 1979, Novak et. al, 1998) and an advanced simulation technique, Importance Sampling (Melchers, 1989). Results obtained from the assessment are analyzed on the basic of convergence and sensitivity analysis.

8.2. Methodology

Since the concrete chloride induced corrosion that involves parameters of high natural variation, the probabilistic assessment is applied in order to evaluate the risk of corrosion initiation, and the SBRA Marek et al. (2003) is used.

The penetration rate of chloride into concrete is generally modeled as a function of depth and time by using Fick's Second Law of Diffusion:

$$\frac{\partial C(x,t)}{\partial t} = D_c \frac{\partial^2 C(x,t)}{\partial x^2}, \tag{8.1}$$

where:

$C(x,t)$ – the chloride ion concentration at a distance x from the surface of concrete in time t

D_c – effective diffusion coefficient, which characterizes the concrete ability to withstand the penetration of chlorides

Solution for the differential equation (8.1) with boundary conditions (Collepardi, et al., 1972) can be expressed as:

$$C(x,t) = C_0 \left\{1 - erf\left(\frac{x}{\sqrt{4.D_c.t}}\right)\right\} \quad ,(8.2)$$

where:

C_0 – chloride concentration on the exposed surface of concrete (%)

erf – error function

Chloride threshold is the concentration of chlorides at the reinforcement level with sufficient content to initiate the corrosion. It is calculated by percentage of chloride per mass of cementitious material.

Equation (8.2) is widely used for 1D analysis without taking into account of a combined transport of water and chloride ion and influence of a crack (Tikalsky, et al., 2005). Its numerical solution is represented by the following polynomial:

$$C(x,t) = C_0 \left\{1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{14} \frac{(-1)^n \left(\frac{x}{\sqrt{4D_c t}}\right)^{2n+1}}{n!(2n+1)}\right\} \tag{8.3}$$

with n is the number of members of a polynomial.

For chloride induced corrosion, the limit state equation can be written as:

$$RF_t = C_{th} - C(x, t), \tag{8.4}$$

where:

RF_t – reliability function
C_{th} – chloride threshold

The input parameters may be described as single value or treated as random variable parameters in order to address random character of the studied problem. The probabilistic approach is adopted herein with the help of three Monte Carlo-based methods described below:

Monte Carlo method is based on a process of repeated random sampling to obtain numerical results. Its methodology includes: creating the sample, running the model and analyzing the data. It is worth noticing that the main advantages of Monte Carlo approach are: (i) simplicity of numerical implementation for complex cases; (ii) reliability and accurateness. Its drawback, however, is computation burden related with complex problems and large number of samples.

To compensate for above mentioned disadvantage of simple Monte Carlo (MC) method, a variance reduction technique such as Latin Hypercube Sampling (LHS) is necessary to have reasonable results within reasonable time. This is a special type of MC simulation approach, using the stratification of the theoretical probability distribution function of input random variables (McKay M.D. et al. 1979). The space of each random variable is divided into subsets of equal probability and outcome is generated from those subsets (Novak D. et al. 1998). Using this stratified simulation technique, reliability analysis is just repeated through sufficient repetitions lower than those of the simple MC. Simulations of random variable inputs are carried out systematically through the distribution function and are randomly mixed to secure their random interaction. This technique has better estimates of reliability function statistical parameters. Thus, expressing reliability level in the form of index β is suitable. Also the Cornell probability of failure may be computed from the standard normal distribution with the mean value and the standard deviation.

Importance Sampling (IS) is another type of advanced simulation techniques, focusing in the failure region and hence obtaining faster convergence to the true probability of failure (Melchers R.E. 1989). In this advanced technique, simulations of random variable inputs are carried out randomly and proportional to the suitable distribution function Y_i^* and the reliability level is described as:

$$p_f \approx \frac{1}{N} \sum_{i=1}^{N} g(Y_i^*) \prod_{j=1}^{k} m_{i,j}\left(X_{i,j}^*\right), \tag{8.5}$$

where:

N	–	number of simulations
k	–	number of variables
$g(Y_i^*)$	–	reliability margin value based on suitable distribution Y_i^*
$m_{i,j}(X_{i,j}^*) = \frac{X_{i,j}}{X_{i,j}^*}$	–	ratio between original probability of $X_{i,j}$ occurrence and occurrence from suitable distribution $X_{i,j}^*$

The initiation of corrosion due to chloride penetration is considered as a serviceability limit state (SLS) and it does not immediately results in extreme consequences to the structure. It's necessary that designed reliability well matches with the actual performance of structure throughout the expected service life. Therefore, the computed probability of corrosion initiation will be compared with target probability. A failure probability of 10^{-1} for SLS are recommended (fib, 2006). For a life-span of 50 years, target probability of corrosion initiation of 25% is proposed (Tikalsky 2003) and adopted in (Konečný, 2007).

To make a comparative assessment of durability analysis as targeted, simulations were carried out with three different simulation techniques: simple MC, LHS and IS with number of samples (N) up to 100.000 for each in selected points of time: 10 years, 20 years, 30 years, 40 years and 50 years. The process is repeated 30 times in order to address the variability of evaluated probability of failure itself. The resulting probability is expressed with mean value and expected range (mean ± 3×standard deviation). Thus, if normal distribution is considered for the description of variability of probability of failure estimation then 99.73 % of resulting probabilities shall be found within expected range.

8.3. Typical durability analysis

The input parameters for the sample simulation of durability analysis are given in Table 8.1. Results from simple MC technique in the case of N=10.000 and t=10 years are shown in Fig. 8.1. The figure contains histograms of chloride concentration at reinforcement level, chloride threshold and reliability function. Also 2D scatter of concentration at reinforcement level and chloride threshold are available on the Fig. 8.1.

Table 8.1. Input data for probability analysis

Parameters	Range/Value	Probability density function	Reference documents
Surface chloride content, C_0 (% weight of cement)	0.21 – 1.63	Histogram	Weyers, et al., 1998
Reinforcement depth, x (m)	0.04 – 0.11	Histogram	Sohanghpurwala and Scannell, 1994
Effective diffusion coefficient, D_c ($m^2/s*10^{-12}$)	0 - 25	Histogram	Sohanghpurwala and Scannell, 1994
Critical chloride content, C_{th} (% weight of cement)	0.09 – 0.51	Histogram	Darwin et al., 2009

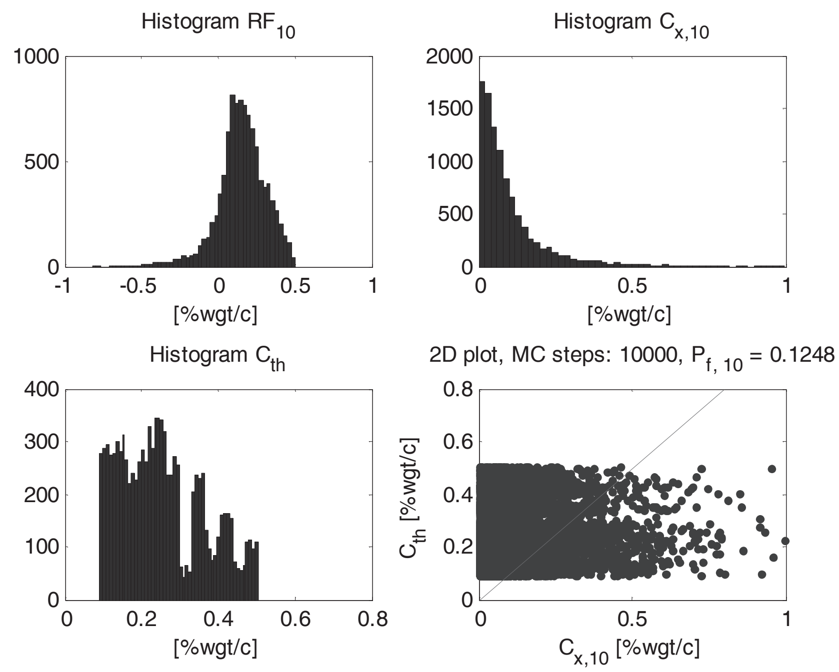

Fig. 8.1. Results from one sample simulation with simple MC technique

And Fig. 8.2 presents probability of corrosion initiation for one simple MC simulation with N=10.000 and chloride exposure time up to 50 years.

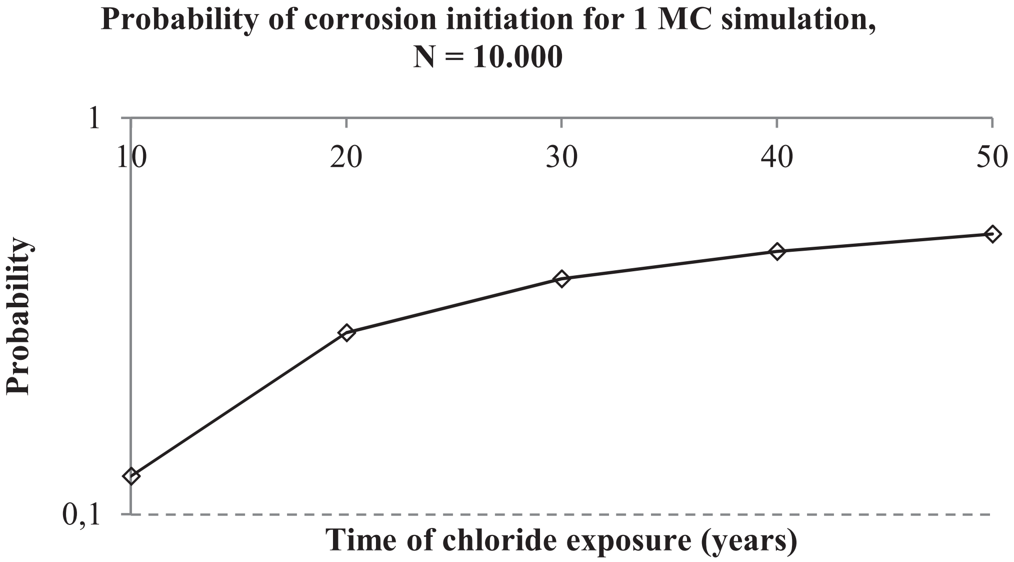

Fig. 8.2. Probability of corrosion initiation for one simple MC technique during simulated life span.

8.4. Numerical experiments

If the MC simulation with 10.000 steps is repeated 30 times then the statistics of probability of corrosion initiation for selected time are computed (mean, standard deviation). The process is also repeated for thousand and hundred thousand simulation steps. Thus, the effect of number of simulations on the probability of corrosion initiation is studied and clearly shown in Fig. 8.3, Fig. 8.4 and Fig. 8.5.

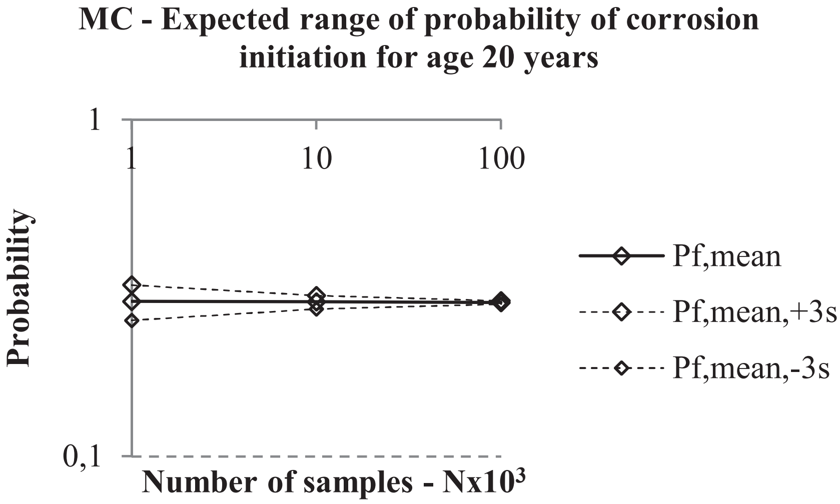

Fig. 8.3. The influence of number of simulations to the probability of corrosion initiation with simple MC technique at time 20 years

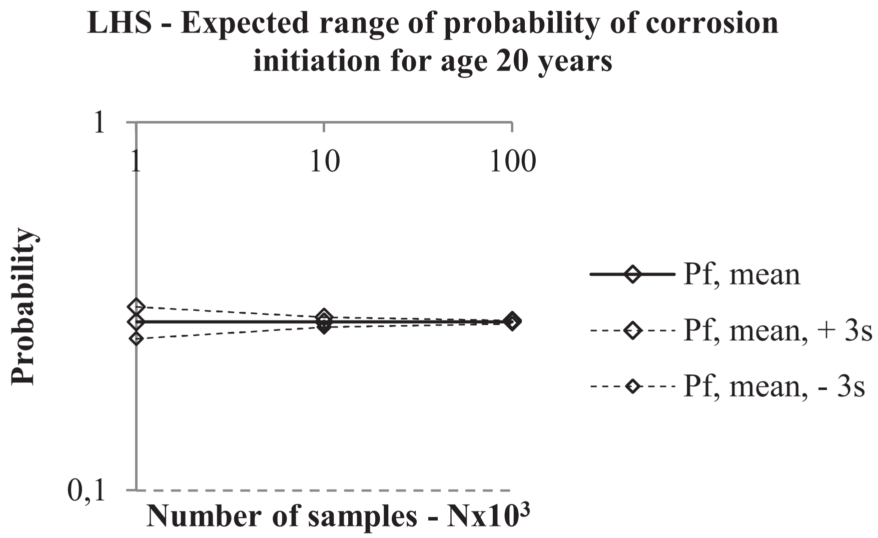

Fig. 8.4. The influence of number of simulations to the probability of corrosion initiation with LHS technique at time 20 years

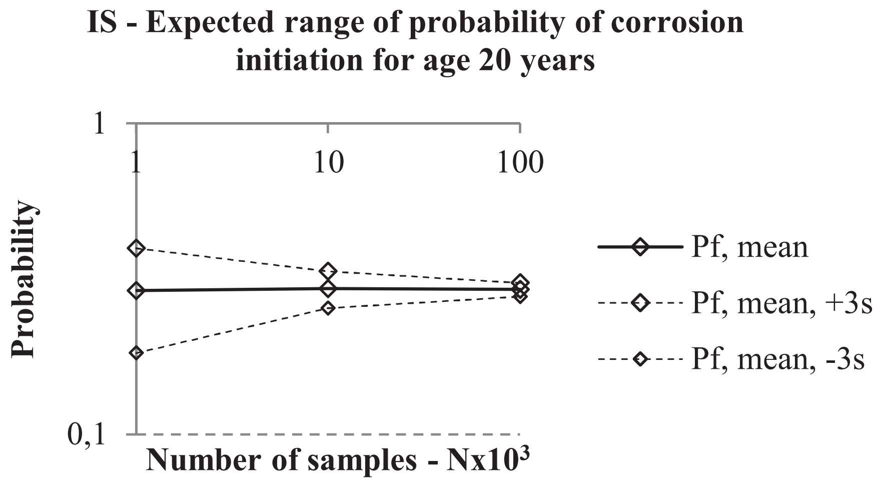

Fig. 8.5. The influence of number of simulations to the probability of corrosion initiation with IS technique at time 20 years

In addition, it is important to note that durability of RC structures vary significantly over the passage of time. The studied RC slab is not an exception. This variation is sure to make the durability of the slab reduce over the time and then its functional performance. In contrast, the probability of corrosion initiation increases with time. Therefore, the variation over the time of the probability of corrosion initiation is also studied here for all three simulation techniques (MC, LHS and IS) and typically described in Fig. 8.6, Fig. 8.7 and Fig. 8.8, respectively.

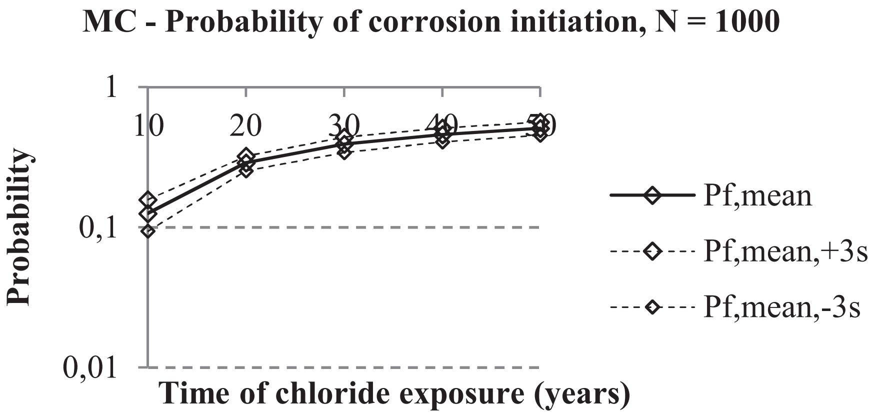

Fig. 8.6. The variation over the time of the probability of corrosion initiation with simple MC technique and N=1000

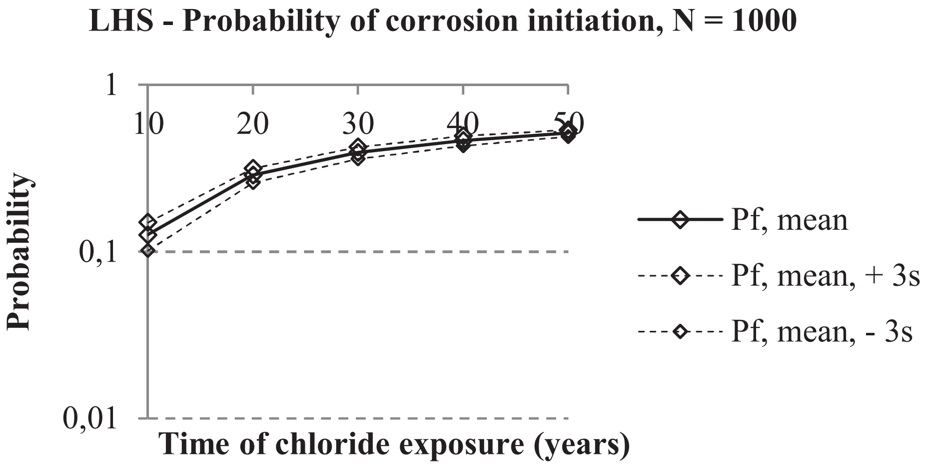

Fig. 8.7. The variation over the time of the probability of corrosion initiation with LHS technique and N=1000

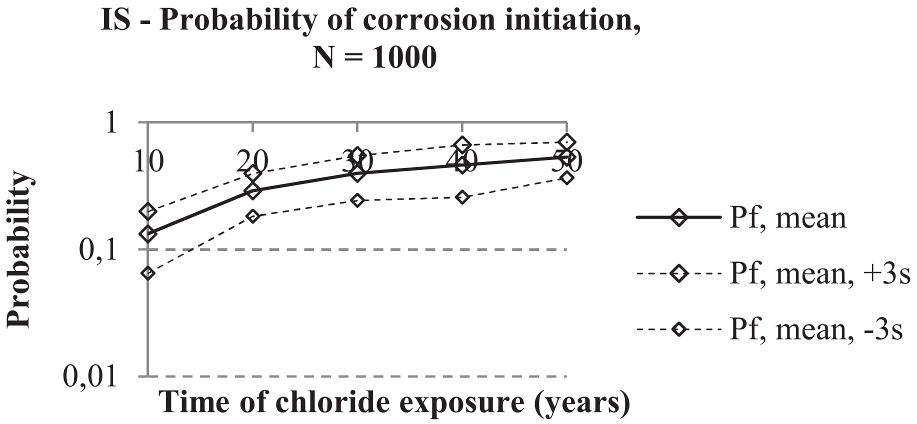

Fig. 8.8. The variation over the time of the probability of corrosion initiation with IS technique and N=1000

8.5. Results

Total of 1350 simulations for mentioned ideal RC slab using three different techniques were done. Results from all three simulation techniques are combined and compared to have a clear assessment of the durability of the slab under chloride penetration. The assessment is carried out through two main parameters: the probability of corrosion initiation and coefficient of variation of the probability of corrosion initiation. The probability of corrosion initiation and its confidence bound for each value of time are depicted in Fig. 8.9, Fig. 8.10, Fig. 8.11, Fig. 8.12 and Fig. 8.13. The coefficient of variation of the predicted probability of corrosion initiation for each value of time is presented in Fig. 8.14, Fig. 8.15, Fig. 8.16, Fig. 8.17 and Fig. 8.18.

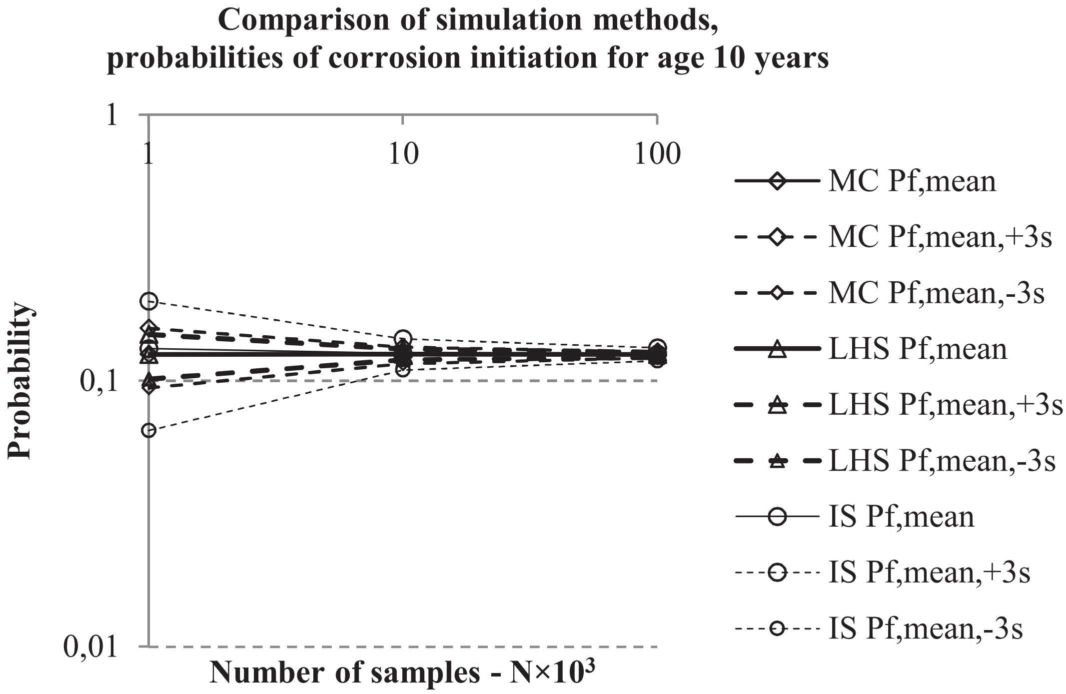

Fig. 8.9. Estimated probability of corrosion initiation at time 10 years

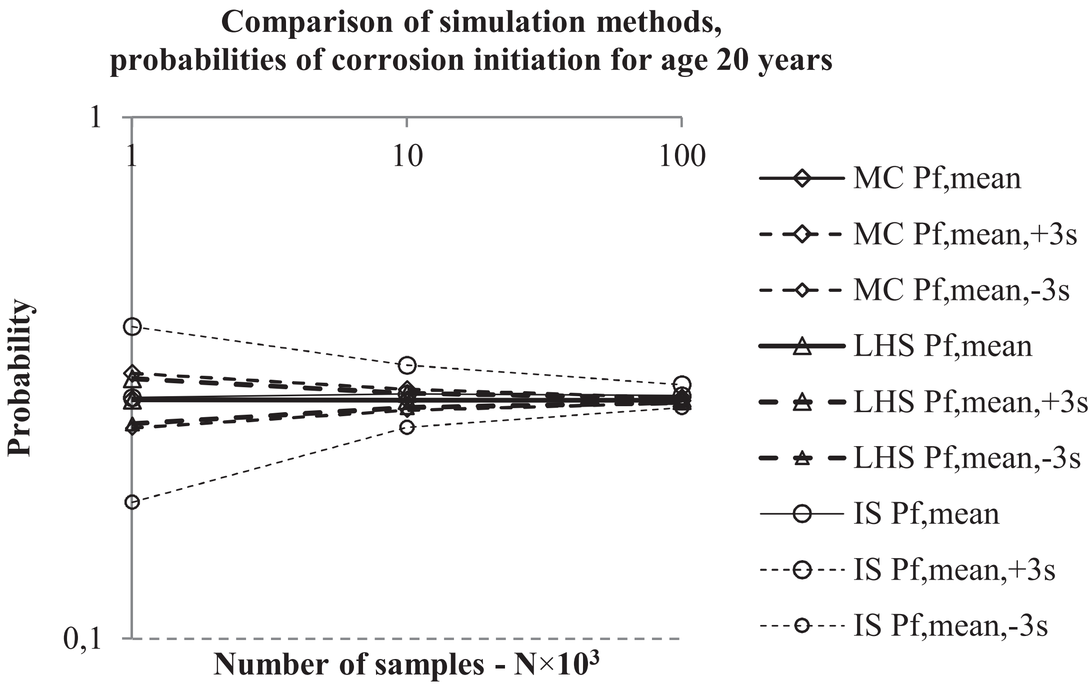

Fig. 8.10. Estimated probability of corrosion initiation at time 20 years

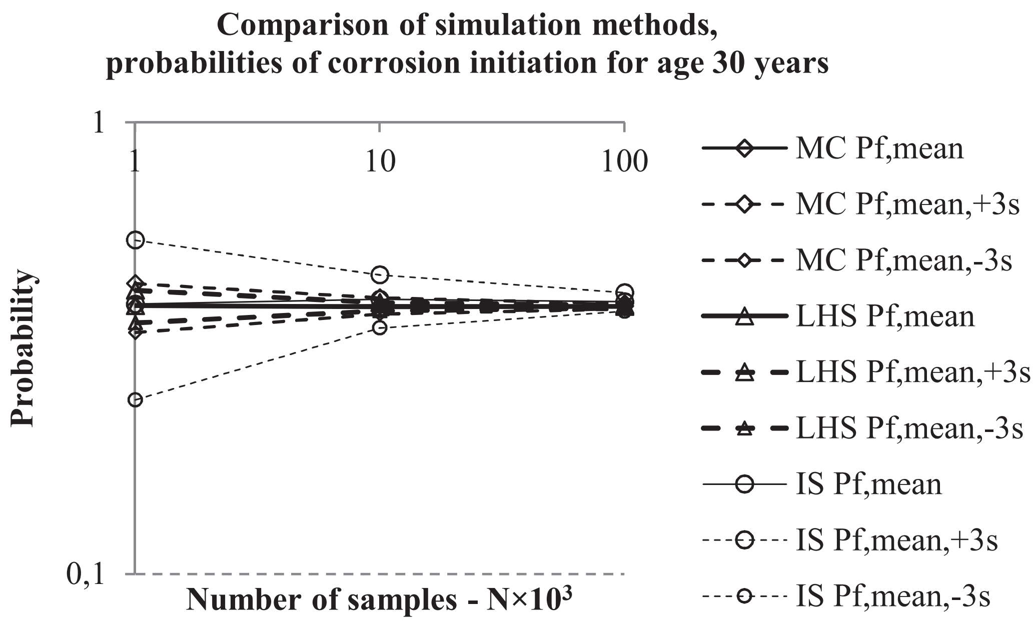

Fig. 8.11. Estimated probability of corrosion initiation at time 30 years

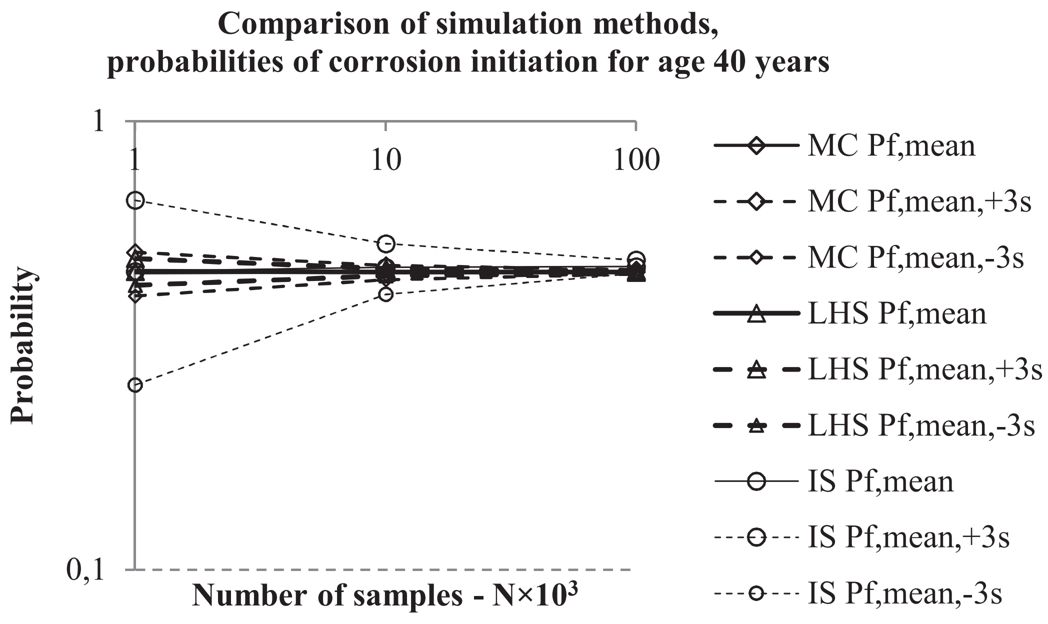

Fig. 8.12. Estimated probability of corrosion initiation at time 40 years

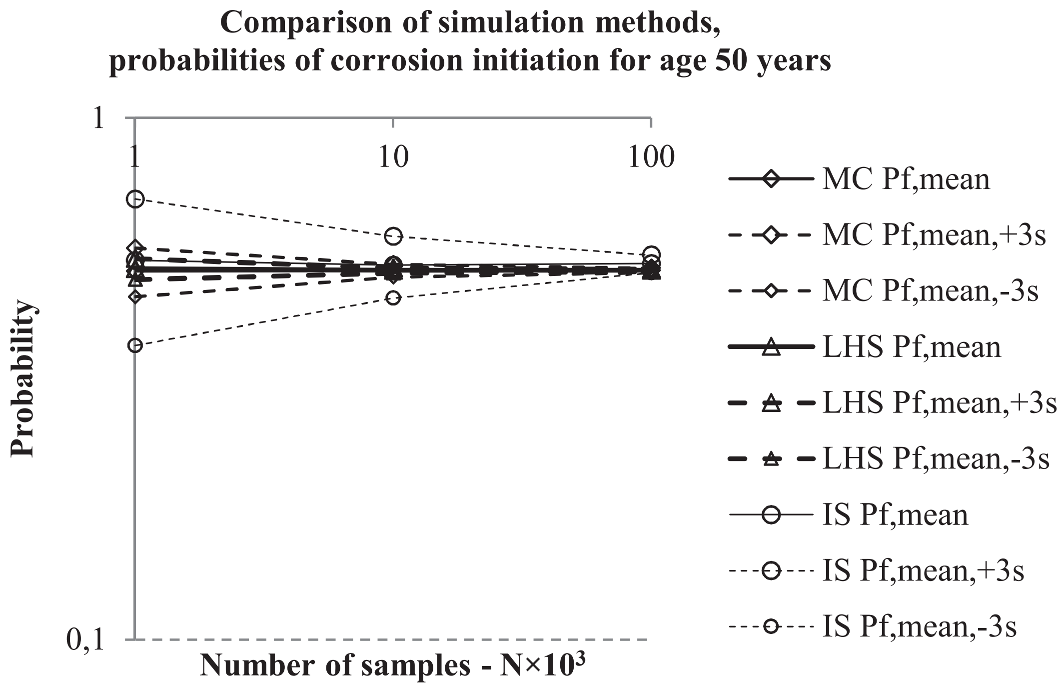

Fig. 8.13. Estimated probability of corrosion initiation at time 50 years

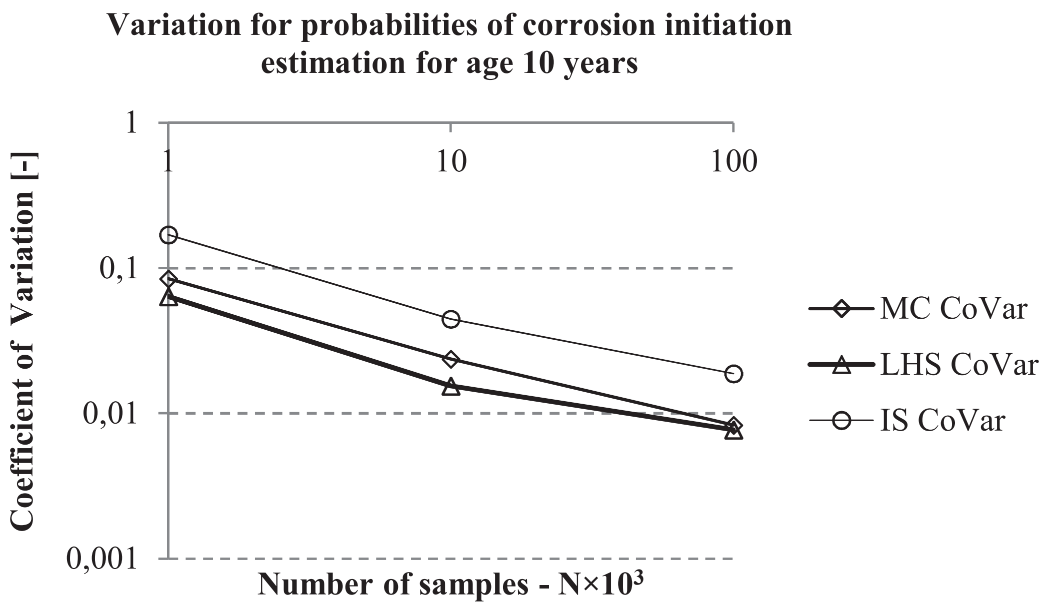

Fig. 8.14. Variation of estimated probability of corrosion initiation at time 10 years

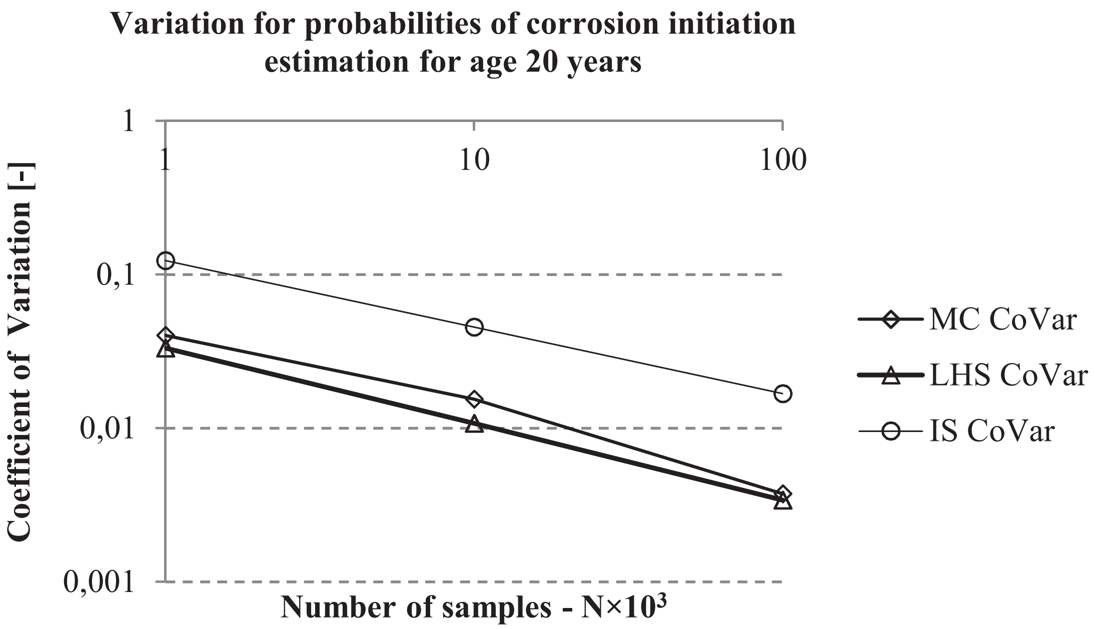

Fig. 8.15. Variation of estimated probability of corrosion initiation at time 20 years

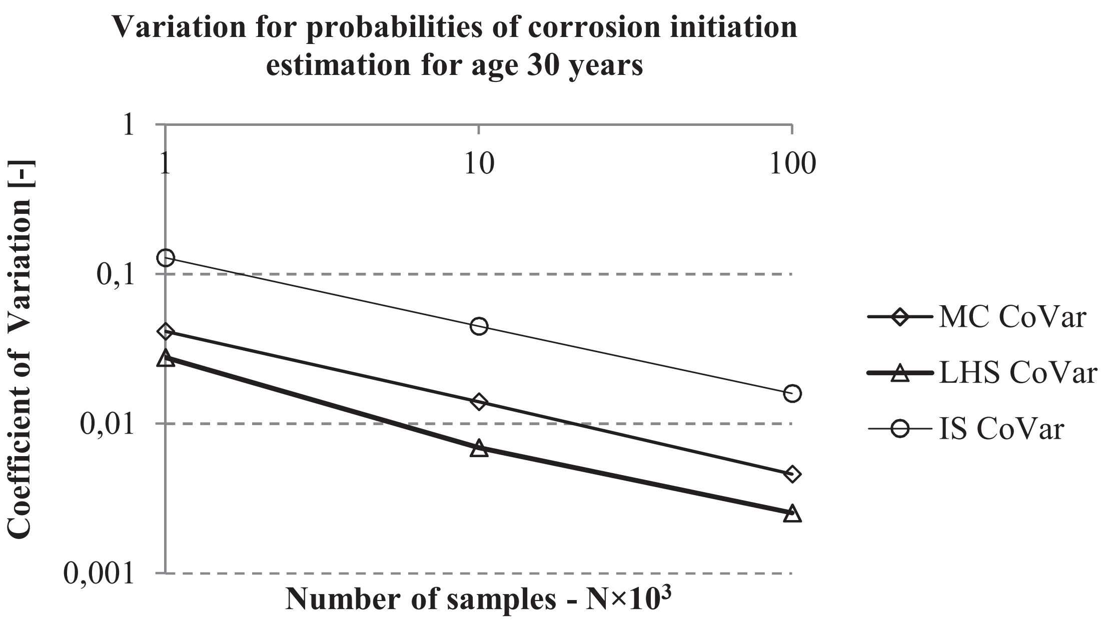

Fig. 8.16. Variation of estimated probability of corrosion initiation at time 30 years

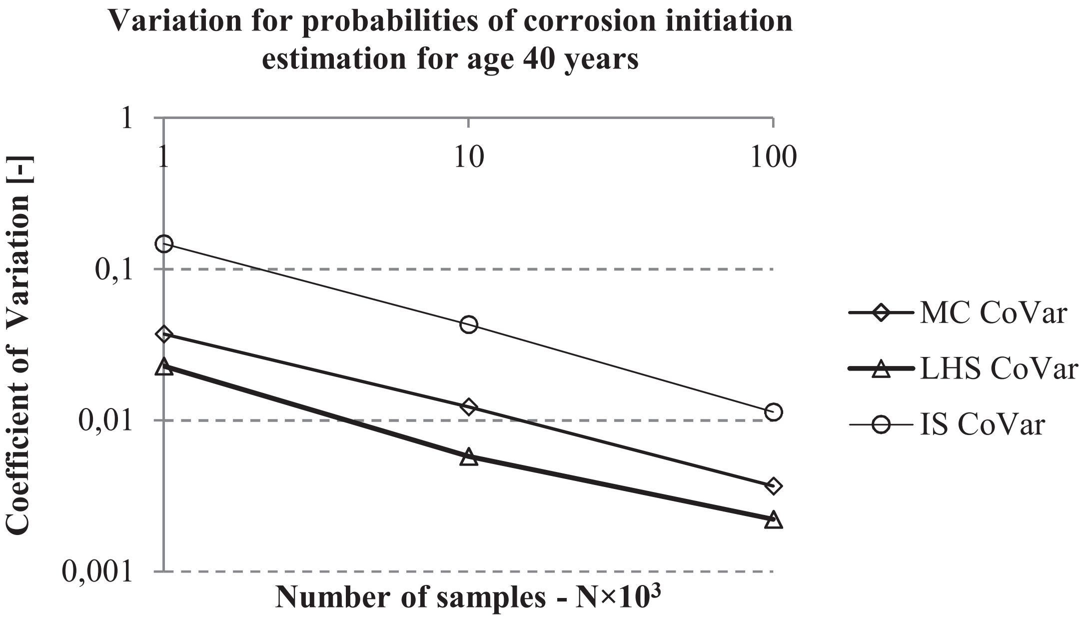

Fig. 8.17. Variation of estimated probability of corrosion initiation at time 40 years

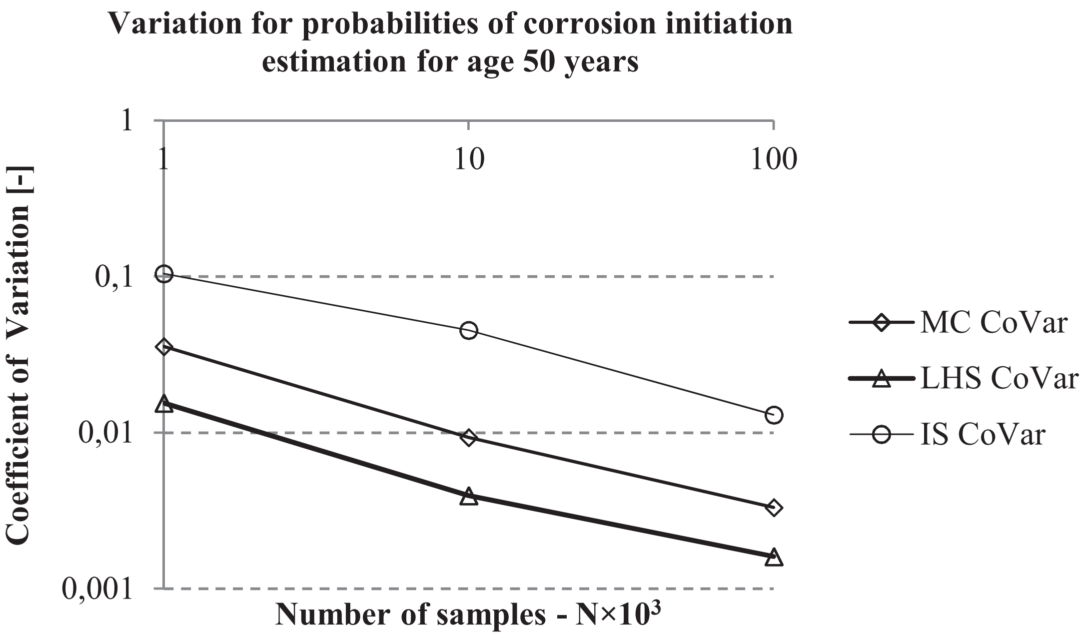

Fig. 8.18. Variation of estimated probability of corrosion initiation at time 50 years

8.6. Summary and conclusions

The probability-based durability assessment of a RC ideal slab with respect to ingress of chloride was carried out through using the scheme of SBRA in which three different MC simulation based techniques were applied, including simple MC, LHS and IS. Random input parameters were introduced via bounded histograms. Fick's Second Law of Diffusion was implemented to model the penetration rate of chloride into concrete. A limit state equation was used to describe chloride induced corrosion. Durability analysis was done by numerical experiment with a total 1350 simulations. Age of the slab and number of simulation samples were the two main parameters of this study.

The values of probability of corrosion initiation resulted from three simulation techniques are very close together on which those of from simple MC and LHS techniques are almost the same while results from IS technique are more scattered, especially with N=1000.

Estimated probability of corrosion initiation at age 10 years is around 12.5% in case of simple MC and LHS techniques while that of IS technique is 12.7%. All these values are larger than the targeted probability, 10%, as recommended by Lausanne for SLS. Furthermore, at the age of 50 years, predicted probability of corrosion initiation is about 51% with simple MC and LHS techniques and approximately 52.3% with IS technique, doubled than the target probability of 25% proposed by Tikalsky.

It's also remarkable from the results that probability of corrosion initiation rapidly converged as number of simulation samples increase and this is true for all three simulation techniques.

In all 5 considered points of time (10 years, 20 years, 30 years, 40 years and 50 years), variation of probability of corrosion initiation is ranging circa from 0.1 to 0.01 and is similar in case of simple MC and LHS techniques. While in case of IS technique, the variation coefficient is circa 3 to 5 times higher comparing to simple MC and LHS approaches. In addition, it is substantial noted that variation rate of the probability when using IS technique sharply increase with age of the slab. The difference of results in IS technique from the other two techniques can be explained by the facts that with this simulation technique, the actual simulation does not go through actual distribution, it goes through a uniform one.

It will be valuable to continue testing with more complicated models such as 2D chloride ingress one. Also combination of durability, serviceability and safety criterion in case of RC beam, column and frame in chloride exposure will be of special interest.

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