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10. Selected problems of the foundation slab under the residential building

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10.1. Introduction

Building structures always rest on the eareth´s surface, so it is necessary to analyze these structures in interaction with subsoil. The use of FEM introduced a lot of advantages to modeling of structures in interaction with the subsoil. It allows the inhomogeneous subsoil, anisotropy to be implemented into the calculation. Such a slab can be modelled in different ways. One possible way is the one-parametric – Winkler´s model of subsoil. For such a model there are various methods of calculating the soil stiffness shown in the chapter 10.2.1. The foundation slab was modeled in ANSYS software used the Shell63 element, which allows including the ground mass into the analysis. In the case of modeling of foundation slabs on elastic half-space, it is necessary to model a quite large portion of ground mass, outside the active depth of deformation. For assessing the structure, international standards and regulations, together with the National Annexes, have been used. Eurocodes also permit assessment of building structures based on probabilistic analysis. Numerous authors have been dealing with probability analyses: Haldar & Mahadevan 2000, Marek at al. 2003, Krejsa & Kralik 2015, and others.

10.2. Foundation slab in interaction with subsoil

An inseparable part of a building structure is the ground mass on which the structure is built. This massive area is infinite, however in modeling of structures in interaction with the subsoil it must have its limits. It is therefore important to choose the right mathematical and material model, which describes the background of the problem. The elastic foundation can be modeled with these types of models:

- Winkler subsoil model (One-parametric model),
- Pasternak subsoil model (Two-parametric model),
- Boussinesq model (Theory of elastic half space).

10.2.1. Winkler subsoil model

Winkler model is the first model of the subsoil, based on the hypothesis of setting coefficient (coefficient of subgrade stiffness), which assumes that at any point of the subsoil there is the load $p(x,y)$ directly proportional to deflection $w(x,y)$ in this point, but it is not dependent on the deflection at the other points. There is drawback in the instability of the setting coefficient (subgrade stiffness) k which depends not only on the material properties of the subsoil, but on the dimensions and the shape of the foundation structure as well. A slab on an elastic foundation is controlled by partial differential equation (10.1).

$$D\,\nabla^2\nabla^2\,w(x,y) + k\,w(x,y) = p(x,y) \tag{10.1}$$

Where:

D – slab constant - see eq. 10.2,
k – elastic foundation stiffness,
$p(x,y)$ – contact stress,
$w(x,y)$ – deflection.

$$D = \frac{E\,h^3}{12\,(1-\nu^2)} \tag{10.2}$$

Where:

E – modulus of elasticity of the material of slab,
h – thickness of slab,
ν – Poisson´s ratio,

Contact stress $p(x,y)$ in point can by calculated by the eq. (10.3) and is independent on the deflection at another point.

$$p(x,y) = k\,w(x,y) \tag{10.3}$$

The pressure in any spring is directly proportional to its pressing and is independent from the other springs. Despite the defect of this model, Winkler's hypothesis can be used very successfully for a material with a low shear stress.

The elastic foundation stiffness can be calculated by individual authors:

Gorbunov - Posadov $$k = \frac{2E}{\pi(1-\nu^2)}\frac{1}{R} \tag{10.4}$$

Pasternak $$k = \frac{E}{H(1-\nu^2)} \tag{10.5}$$

Vlasov-Leontiev $$k = \frac{E_{oed}}{H(1-\nu^2)} \tag{10.6}$$

Barvašová $$k = \frac{E}{H(1-\nu)} \tag{10.7}$$

Prakasm
$$k = \frac{1.13\,E}{(1-\nu^2)}\frac{1}{\sqrt{A}} \tag{10.8}$$

For a rectangle slabs ($2b < 2a$)
$$k = \frac{E}{16\,a\,(1-\nu^2)}\left[3.1\left(\frac{a}{b}\right)^{0.75} + 1.6\right] \tag{10.9}$$

Formulas see eq. (10.4) - (10.7) may be applied to soil foundation, where the relatively thin layer H lies on a hard base. For the foundations of larger planar dimensions as the foundation slabs the relations see eq. (10.8) - (10.9) are useful. In the case where the arrangement of the foundation structure s is available, the relationship see eq. (10.10) can be used to calculate
$$k = \frac{p}{s}, \tag{10.10}$$

where

p – contact stress,

s – setlement.

10.2.2. Pasternak subsoil model or two-parametric model

Two-parametric model of the subsoil is defined by Pasternak so that the contact stress p is defined according to the deflection surface of $w\,(x, y)$ and coefficients C_1 - coefficient of friction in the vertical direction (N / m^3), and C_2 - coefficient characterizing the shear spreading of the effects of the load (N / m) obtained using energy principles. This model allows to take into account deformations beyond the foundation.
$$D\,\nabla^2\nabla^2\,w(x,y) + C_1\,w(x,y) - C_2\left[\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right] = p(x,y) \tag{10.11}$$

10.2.3. Boussinesq model – theory of elastic half space

Elastic half-space is another type of mathematical - material model of the elastic subsoil. The theory of elastic continuous half-space implies that the background forms a continuous, homogeneous and perfectly elastic body of an infinitely large size, limited from above by the plane on which lies the underlying building structure. Sometimes it is called the Boussinesq soil model. The model is characterized by two material constants derived from experimental measurements (E - modulus of elasticity and ν - Poisson´s constant). Numerical methods are primarily used to solve contact problems (subgrade model - elastic half-space) to appropriately extend the closed form solutions while offering the possibility of more realistic capturing the interaction characteristics with the subsoil building structure for different boundary conditions. The most famous is the FEM. Theoretical assumptions of Boundary Element Method - BEM and Finite Element

Method - FEM in interaction were dealt by Zienkiewicz & Taylor 2000, Jendželovský & Baláž 2014, Sumec et al. 2010, Kotrasova & Kormanikova 2016, Klucka et al. 2014 and others.

Elastic foundation is modeled by spatial (3D) finite elements. Finite elements of different shapes can be used for the modeling. The most commonly used form are solid block elements with 8 nodes, and three displacements at each node. In modeling of elastic half-space it is necessary to create a sufficiently large volume of grown earth, which depends on supports, boundary conditions. The support may be rigid, flexible or by infinite finite elements.

For the analysis of interaction foundation sla with the subsoil (static bonds on the contact surface) two basic models can be defined: computational model of bilateral bonds - continuous model, calculation model with unilateral bonds - discrete model.

10.3. Probability analysis

To determine the reliability of probabilistic methods, first the performance criteria are defined on the basis of functional relationship between the first *n* input variables, called bases random variables *Xi*, where *i = 1, 2, ..., n*. This relationship is called a function of reliability (security, usability, feature or function, or function failure reliability reserves) and is marked as see eq. (10.12)

$$F_s = RS = g\ (X_1, X_2, \cdots, X_n). \tag{10.12}$$

A functional dependency $g\ (X_1, X_2, \cdots, X_n)$ is a computational model that is based on simplifying assumptions and represents the idealization of physical reality.

Failure functions $g\ (X_i)$ express conditions of probability (reserve) and can be represented as a function of stochastic parameters. They may be defined as a simple (e.g. for one section) or as complex structures, for more cross-sections (e.g. all finite-element model).

The reliability function can be also defined as:

$$F_s = RF = R - S \tag{10.13}$$

where:

R – is the resistance of the structure function (yield strength, allowable deflection, stress),

S – is a function of the load (max. stress in the structure, max. deflection).

The function is reliable if $F_s \geq 0$, failure occurs if $F_s < 0$ and $F_s = 0$ is the limit function.

The probability of failure can be simply defined as:

$$P_f = P[R < S] = P[R - S < 0] \tag{10.14}$$

where:

P_f – is generally given by the expression (10.15),

$$P_f = \iint \cdots \iint_{g(X)<0} f_X(X_1, X_2, \cdots, X_n)\, dX_1\, dX_2 \cdots dX_n \tag{10.15}$$

where the function $f_X(X_1, X_2, \ldots, X_n)$ represents the multiple density of probability function for continuous random variables or multiple probability function for discrete random variables, where integration is transferred over the entire area of failure $g(X) <0$.

In deterministic calculation we entered the input parameters as the fixed constants. When we used probability calculation, the input parameters specified in the range were accidental due to inaccuracies in manufacture and the determination of material characteristics. The individual parameters varied according the diagrams.

10.3.1. Response surface analysis design

Response Surface Methods are based on the fundamental assumption that the influence of the random input variables on the random output parameters can be approximated by mathematical function. Hence, Response Surface Methods locate the sample points in the space of random input variables such that an appropriate approximation function can be found most efficiently; typically, this is a quadratic polynomial. In this case the approximation function $\hat{U}$ is described by eq. (10.13)

$$\hat{U} = c_0 + \sum_{i=1}^{NRV} c_i X_i + \sum_{i=1}^{NRV} \sum_{j=1}^{NRV} c_{ij} X_i \,.\, X_j, \tag{10.13}$$

where c_0 is the coefficient of the constant term, c_i, $i = 1,...NRV$ are the coefficients of the linear terms and c_{ij}, $i = 1,...NRV$ and $j = i, ...,NRV$ are the coefficients of the quadratic terms.

To evaluate these coefficients a regression analysis is used and the coefficients are usually evaluated such that the sum of squared differences between the true simulation results and the values of the approximation function is minimized. Hence, a response surface analysis consists of two steps:

- Performing the simulation loops to calculate the values of the random output parameters that correspond to the sample points in the space of random input variables.
- Performing a regression analysis to derive the terms and the coefficients of the approximation function.

The fundamental idea of Response Surface Methods is that once the coefficients of a suitable approximation function are found, then we can directly use the approximation function instead of looping through the finite element model. To perform a finite element analysis might require minutes to hours of computation time; in contrast, evaluating a quadratic function requires only a fraction of a second. Hence, if using the approximation function, we can afford to evaluate the approximated response parameter thousands of times. A quadratic polynomial is sufficient in many cases of engineering analysis (for example, the evaluation of the thermal stress mentioned above). For that evaluation, the Young's modulus and the thermal expansion coefficient both have a linear effect on the thermal stresses, which is taken into account in a quadratic approximation by the mixed quadratic terms. However, there are cases where a quadratic approximation is not sufficient; for example, if the finite element results are used to calculate the lifetime of a component. For this evaluation, the lifetime typically shows an exponential behavior with respect to the input parameters; thus the lifetime results cannot be directly or sufficiently described by a quadratic polynomial. But often, if you apply a logarithmic transformation to the lifetime results, then these transformed values can be approximated by a quadratic polynomial. The ANSYS PDS offers a variety of transformation functions that you can apply to the response parameters, and the logarithmic transformation function is one of them.

10.3.1.1. Central composite design sampling

A central composite design consists of a central point, the N axis point plus 2^{N-f} factorial points located at the corners of an N-dimensional hypercube. Here, N is the number of random input variables and f is the fraction of the factorial part of the central composite design. A fraction $f = 0$ is a called a full factorial design, $f = 1$ gives a half-factorial design, and so on. The PDS gradually increases the fraction f as you increase the number of random input variables. This keeps the number of simulation loops reasonable. The fraction f is automatically evaluated such that a resolution V design is always maintained. A resolution V design is a design where none of the second order terms of the approximation function are confined with each other. This ensures a reasonable accuracy for the evaluation of the coefficients of the second order terms.

The locations of the sampling points for a problem with three random input variables is illustrated below, see Fig. 10.1.

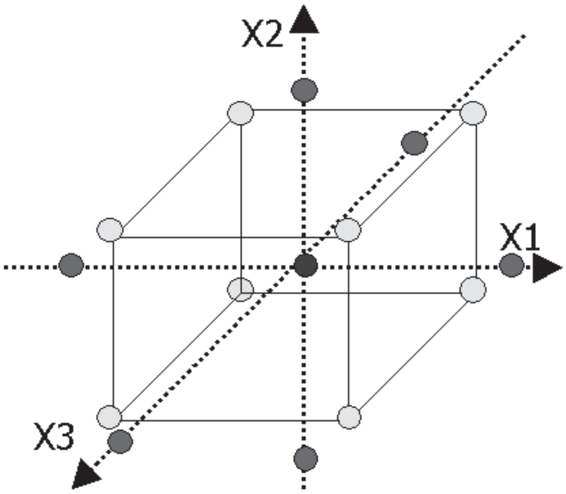

Fig. 10.1. Locations of sampling points for problem with three input variables for CCD

The number of sample points (simulation loops) required for a central composite design as a function of the number of random input variables is given in the table, see tab. 10.1.

Table. 10.1. The number of simulation loops required for a central composite design

Number of random input variables	1	2	3	4	5	6
Number of coefficients in a quadratic function (with cross-terms)	3	6	10	15	21	28
Factorial number *f*	N/A	0	0	0	1	1
Number sample points (simulation loops)	N/A	9	15	25	27	45

10.4. Analyses of the foundation slab under residential building

10.4.1. Deterministic approach

The analysed foundation slab is located under residential building. It is made of concrete of the class *C25/30* with material properties as follows: $E_x = 31\ GPa$, Poisson's ratio $\nu = 0.2$, size *12 x 12 m*, with thickness *0.6 m*. The foundation slab is loaded with singular forces from columns $F = 914 - 3342\ kN$ according to the loading area, see Fig.10.2.

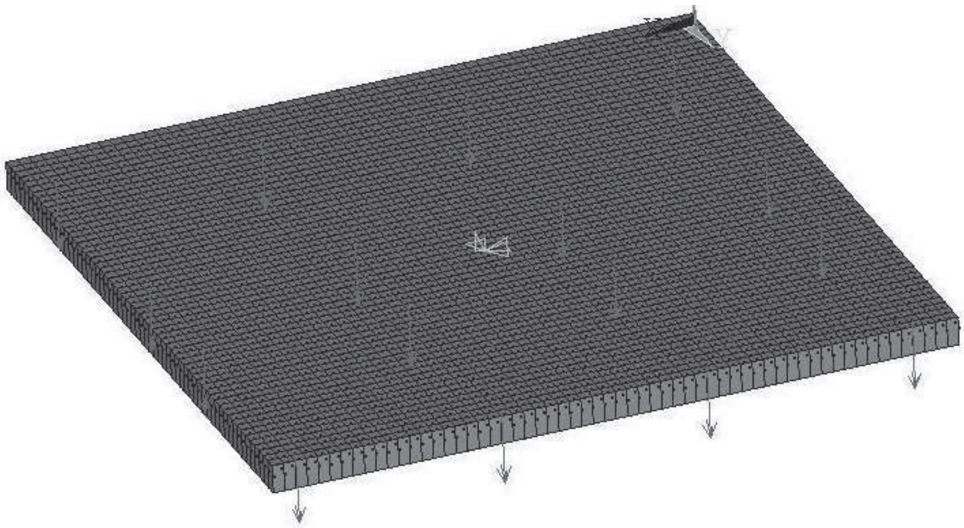

Fig. 10.2. Model of the foundation slab

The foundation slab is rested on a layered subsoil:

- *0 – 1.5 m* from the foundation gap *G5* – clayey gravel, dense
- *1.5 – 9 m* *G2* –Poorly graded gravel
- *9 – 14 m* *F8* – clay with medium plasticity.

The stiffness of the subsoil was determined as $k = 46\ 000\ kN/m^3$ according to formula (10.10). The easiest way to model a slab on en elastic foundation is to use SHELL 63 elements, where it is possible to introduce the stiffness coefficient of the subgrade via the EFS command.

Final deflection of the foundation slab rested on layered subsoil from the deterministic approach is shown in Fig. 10.3.

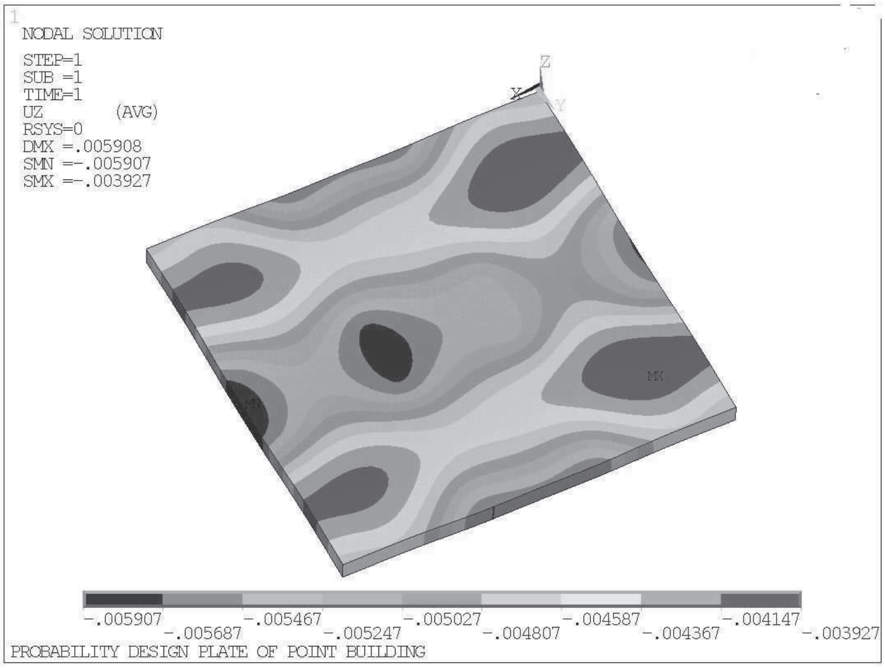

Fig. 10.3. Deflection of the foundation slab from the deterministic analysis

10.4.1. Probabilistic approach

The individual input parameters in the probabilistic analyses are varied according to Tab. 10.2. Change of geometric properties of the foundation slab was defined via thickness *H* and via a change in thickness of the foundation slab - *Hvar_*. The stiffness of the slab was determined via Young´s modulus *EX* and via variable factor *Evar_*., see Fig.10.4. Stiffness of the elastic subsoil was defined via *k* and through a variable factor *KKvar_*. The load was determined through *F* and via variable *Fvar_*.

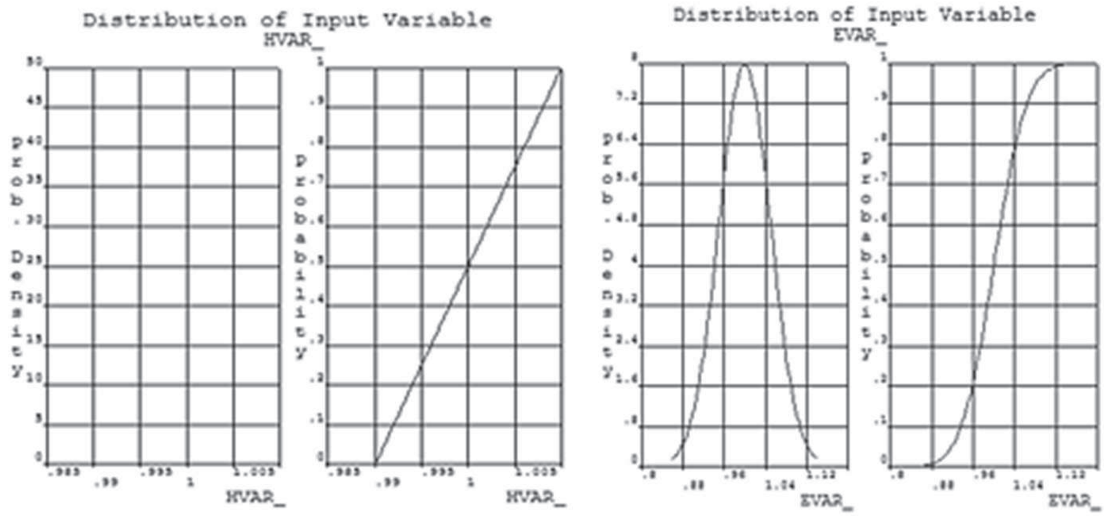

Fig. 10.4. Histogram of input variable *Hvar_* and *Evar_*

Table. 10.2. Specification of the random input variables

Name	Charact. value	Value	Variable Param.	Histogram	Mean or Min. value	St. Dev. or Max.value
Concrete slab						
Young´s modulus	EX [kPa]	3.1E+07	Evar_	GAUS	1	0.05
Poisson´s ratio	mi [-]	0.2	mivar_	GAUS	1	0.05
Thickness	H [m]	0.5	Hvar_	UNIF	-0.01	+0.01
Soil						
Elastic foundation stiffness	k [kN/m3]	4.6E+4	KKvar_	GAUS	1	0.05
Load						
Forces	F1 – F4 [kN]	914-3342	Fvar_	LOG1	1.0	0.1

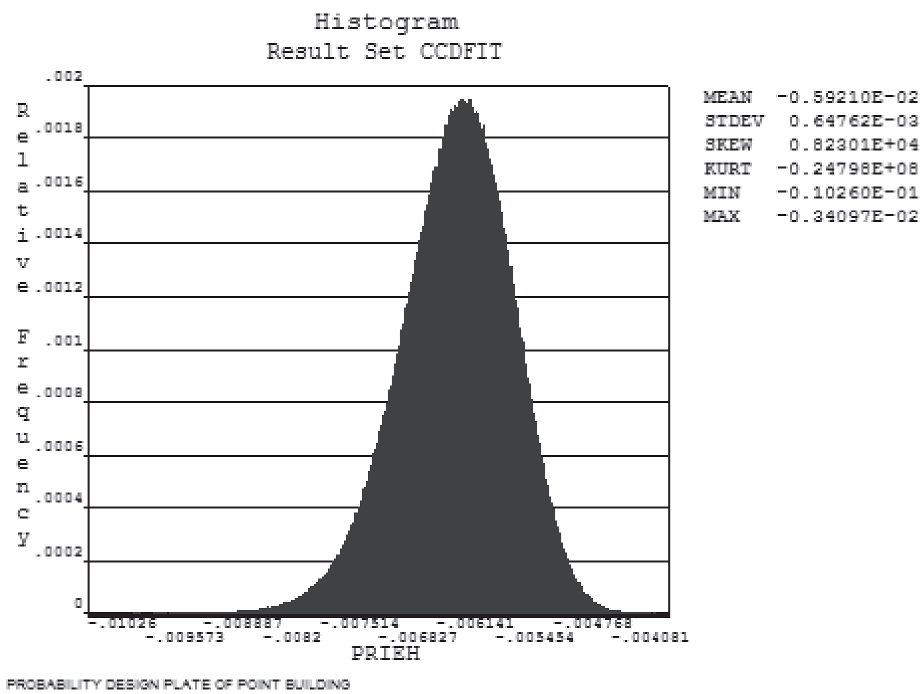

Fig. 10.5. Histogram of max. deflection PRIEH

Resulting from variability of input quantity 25 simulations on the base of RSM method were performed. The probability of exceeding the limit deflection of the foundation slab was calculated from five millions Monte Carlo simulations for 25 simulations of approximation method RSM on the structural FEM model, see Fig. 10.5.

According to the CDF - Cumulative Distribution Function) we can determine probability of the corresponding parameter PRIEH (the maximum value of deflection). The probability that max deflection is less than – 7.0 mm, representing that the design is at 5.6 e-2 unreliable.

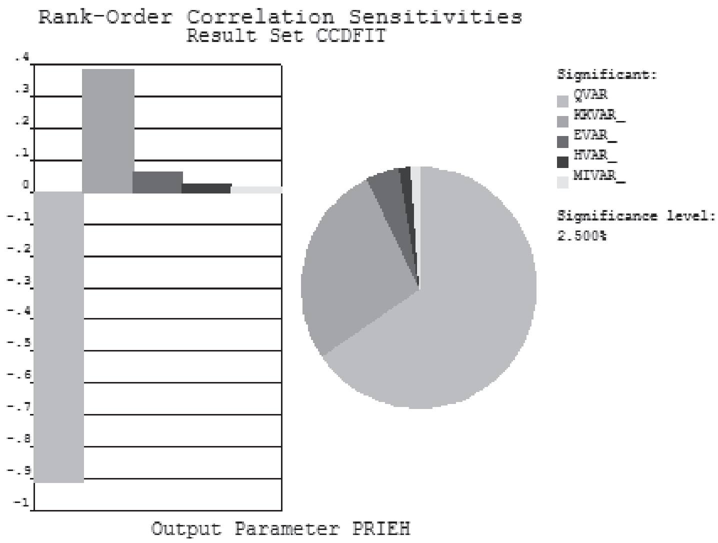

Fig. 10.6. Rank-order correlation sensitivities of output variable PRIEH

The evaluation of the probabilistic sensitivities is based on the correlation coefficients between all random input variables and a particular random output parameter. Either Spearman rank order correlation coefficients or Pearson linear correlation coefficients may be used based on user's specifications. To plot the sensitivities of a certain random output parameter, the random input variables are separated into two groups: those that are significant (important) and those that are insignificant (not important) for the random output parameter. The sensitivity plots will only include the significant random input variables, see Fig.10.6.

10.5. Conclusions

The main aim of this chapter was the deterministic and probabilistic analysis of the foundation slab under the residential building rested on layered subsoil. It was modeled in software Ansys. The layered subsoil was included in the calculation using the stiffness coefficient of the subgrade. From deterministic analysis max. deflection of the foundation slab rested on the layered subsoil was -5.9 mm. After deterministic analysis probability analysis was performed. The goal was to determine the probability of exceeding the deflection limit in the foundation gap by 6 variable input parameters. The probability of failure is equal to 5.6e-2 in the case of deflection limit -7.0 mm (determined by the investor), using 5 million Monte Carlo simulations for 25 cycles of the RSM approximation method on the FEM model.

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