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12. Generalized maximum tangential stress criterion in double cantilever beam specimens: choice of the proper critical distance

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Abstract: The papers is focused on generalized maximum tangential stress criterion in double cantilever beam specimens. The conducted research programme allowed to find the critical distance between 0.35 and 0.6 mm which seems to be suitable for the specimens with the defined geometry and made of the material under the study.

Keywords: tangential stress, beam, defect, critical distance

12.1. Introduction

When a defect/crack is detected in a structure, the fracture mechanics concept is applied in order to estimate the reliability and/or lifetime of the damaged structure. Generally, fracture mechanics (Anderson, 2004) deals with description of cracks behaviour in various kind of materials where different types of fracture can occur; they are usually classified as the brittle, the elastic-plastic or the quasi-brittle case, see e.g. (Bažant and Planas, 1998; Shah *et al.*, 1995). For each kind of fracture specific approaches have been derived and they should not be applied elsewhere because of their limited validity. For example, the linear elastic fracture mechanics approaches cannot predict correct failure response in the case of elastic-plastic or quasi-brittle fracture. Similarly, the strength theory ignoring the stress concentration at cracks/notches/defects tips and also ductile fracture models are not suitable for failures with a rapid and huge energy release (typical for brittle fracture and in some cases for quasi-brittle fracture).

Nevertheless, sometimes the fracture response does not follow any of the theories suggested for the three kinds of fracture; of course, there exist transition zones between the individual types of fracture, see (Veselý *et al.*, 2017). There are always efforts to find some theory that would enable to cover as many fracture kinds as possible.

One of useful tools how to get more precise results when fracture response is described seems to be utilization of the multi-parameter fracture mechanics concept. This theory is based on the assumption that the stress/displacement field near a crack tip can be expressed by means of a power series (Williams, 1957), Williams expansion (WE), i.e. that not only the first (singular) term controls the crack propagation but also the terms of higher orders are significant. This is especially important for materials where the fracture process occurs not only very close to the crack tip but also in a further distance from the crack tip. Typically, the material behaviour in this region is mostly nonlinear and is not sufficiently explored. Moreover, the size/geometry/boundary effect has to be taken into account, see (Ayatollahi and Akbardoost, 2012; Duan *et al.*, 2007; Karihaloo *et al.*, 2006). The influence of the specimen shape on the mode I fracture strength has been demonstrated recently (Chao *et al.*, 2001; Davenport and Smith, 1993; Khan and Al-shayea, 2000; Kumar *et al.*, 2011; Liu and Chao, 2003; Sun and Qian, 2009).

The multi-parameter concept has been applied to several fracture mechanics tasks (such as near-crack-tip stress field approximation, crack propagation assessment, plastic zone extent estimation, etc.) and its significance has been proved ((Šestáková) Malíková, 2013; Šestáková, 2014; Veselý *et al.*, 2014). Subsequent investigations are introduced in this paper.

Particularly, the topic of the presented work is connected to the interest in investigations of the crack path. It has been observed that although the crack is loaded under pure mode I (when it is expected that it will propagate along the direction of the original crack plane), it sometimes kinks (Ayatollahi *et al.*, 2010; Betegon and Hancock, 1991; Du and Hancock, 1991; Larsson and Carlsson, 1973; Rice, 1974). This phenomenon is often explained by means of the constraint effect, i.e. it is connected to the value of *T*-stress (second/non-singular term of the Williams expansion). Several fracture criteria including some initial terms of the WE have been proposed and it has been observed that taking into account the *T*-stress values can bring more reliable fracture assessment (Aliha *et al.*, 2010; Aliha and Ayatollahi, 2010; Ayatollahi *et al.*, 2015; Ayatollahi and Aliha, 2009; Cotterell, 1966; Pook, 2015; Smith *et al.*, 2001).

In order to show importance of the terms of higher orders of the WE, multi-parameter fracture criteria have been derived and effect of T-stress studied, particularly a Generalized Maximum Tangential Stress (GMTS) criterion (Ayatollahi *et al.*, 2015; Smith *et al.*, 2001) or Average Strain Energy Density (ASED) criterion (Lazzarin *et al.*, 2009).

In this work a multi-parameter form of the Maximum Tangential Stress (MTS) criterion is suggested and tested in order to find the initial crack propagation angle. The numerical analyses are based on the experimental observations on PMMA specimens. The basic geometry proceeds from the Compact Tension (CT) specimens and then several more specimens with larger widths (representing Double Cantilever Beam (DCB) specimens) are suggested, modelled and a parametrical study is performed. The choice of the crucial length parameter (critical distance from the crack tip, where the criterion is applied) is discussed thoroughly.

12.2. Basic terms and theory

The key idea of this paper is the approximation of the stress/displacement field near the crack tip by means of the Williams power series. This theory can be involved in fracture mechanics tasks and assumes that not only the first singular term is important for the stress state in a cracked specimen. Whereas the classical linear elastic fracture mechanics considers only the stress intensity factor as the single-controlling parameter, WE enables to take into account also the higher-order (non-singular) terms. The theory was derived for a crack in an elastic isotropic homogeneous body subjected to an arbitrary remote loading. Then, the stress tensor and displacement vector components for a crack under I+II mixed mode can be expressed as follows:

$$\sigma_{ij} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} f_{I,ij}(\theta, n) + \sum_{m=1}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}-1} f_{II,ij}(\theta, m) \quad (12.1)$$

and

$$u_i = \sum_{n=0}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}} g_{I,ij}(\theta, n, E, v) + \sum_{m=0}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}} g_{II,ij}(\theta, m, E, v) \quad (12.2)$$

The meaning of the symbols is:

i, j – $i, j = \{x, y\}$,

r, θ – polar coordinates (assuming the origin of the coordinate system at the crack tip and the crack faces lying on the negative x-axis),

$f_{I,ij}(\theta, n)$ – known functions corresponding to the stress tensor components and mode I of loading,

$f_{II,ij}(\theta, m)$ – known functions corresponding to the stress tensor components and mode II of loading,

$g_{I,ij}(\theta, n, E, v)$ – known functions corresponding to the displacement vector components and mode I of loading,

$g_{II,ij}(\theta, m, E, v)$ – known functions corresponding to the displacement vector components and mode II of loading,

E, ν	–	material parameters, i.e. Young's modulus and Poisson's ratio,
A_n, B_m	–	coefficients of the higher-order terms; depending on the specimen geometry and boundary conditions.

Both Eq. 12.1 and 12.2 are used in their truncated form in the procedure described in the following sections. First, the approximation of the displacements serves for calculation of the coefficients of the higher-order terms and then the approximation of the stresses is a part of the MTS fracture criterion.

Note, that the well-known stress intensity factor corresponds to the first coefficient of the WE ($K_{\mathrm{I}} = A_1\sqrt{2\pi}$, $K_{\mathrm{II}} = -B_1\sqrt{2\pi}$). Second (non-singular) term corresponds to the in-plane T-stress. Coefficients of the higher orders are not connected to any conventional fracture parameters.

12.3. Over-deterministic method

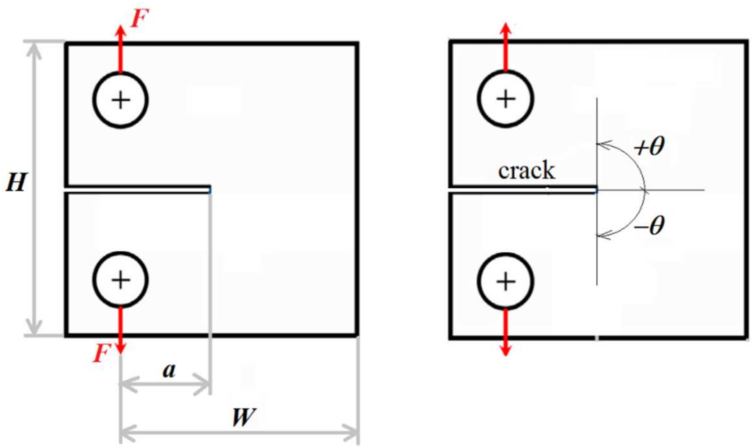
When Eq. 12.1 shall be used for approximation of the crack-tip stress field and application of the multi-parameter MTS criterion, it is necessary to determine the coefficients of the WE. This must be done numerically in the most cases, an over-deterministic method (ODM) is applied in this work, see (Aytollahi and Nejati, 2011) for more details.

The principle of the ODM is relatively simple in comparison to other methods such as hybrid crack element method, boundary collocation method or others that require more extensive and deeper knowledge of the special elements or advanced mathematical procedures (Karihaloo and Xiao, 2001; Knésl, 1994/1995; Su and Fok, 2007; Tong *et al.*, 1997; Xiao *et al.*, 2004).

The ODM procedure based on the formulation of linear least-squares consists in the direct application of Eq. 12.2. Particularly, a finite element (FE) model of the cracked specimen according to the experimental configuration was created to get the nodal solution. Then, the displacements u_x, u_y of a set of nodes at a selected radial distance from the crack tip were further used as inputs for the ODM application. When the FE solution is known in the set of k nodes, a system of $2k$ equations arises. Solution of such a defined system is represented by the coefficients of the individual terms of the WE. Naturally, the crucial idea of the ODM is that the system of equations must be over-determined, i.e. when N mode I coefficients and M mode II coefficients shall be calculated, at least $(N + M)/2 + 1$ nodes with their coordinates and displacements must be taken as inputs for the ODM procedure. Further details on the accuracy, convergence, mesh sensitivity of the method can be found in (Malíková, 2015; Růžička *et al.*, 2017; Šestáková (Malíková), 2013; Šestáková and Veselý, 2013).

12.4. Cracked specimen geometry

As is mentioned in the previous text, several cracked geometries have been chosen and the Compact Tension was the basic one. Additionally, the width of the original specimen was enlarged and several double cantilever beam (DCB) specimen configurations were considered (W = 60, 90 and 150 mm), see Fig. 12.1a.



(a) (b)

Fig. 12.1. (a) Schema of the investigated cracked specimen: $W = 30$ (CT), $W = 60$, 90 and 150 mm (DCB); (b) region where the tangential stress was investigated

The specimen dimensions were considered as follows: specimen height $H = 30$ mm, specimen thickness $B = 10$ mm and specimen width $W = 30$ (CT), $W = 60$, 90 and 150 mm (DCB). The crack length (following the designation introduced in Fig. 12.1a) was always a half of the width, $a = W/2$. The specimen was subjected to tensile loading, the values of the loading force were set up with regard to experimental observations (Ayatollahi *et al.*, 2016). The values of the fracture forces were measured as: $F = 319$, 214, 141 and 87 N (corresponding to the specimen width 30, 60, 90 and 150 mm.

Based on the specified geometry, the specimen was modelled in a FE code, particularly in ANSYS FE code (Ansys, 2016). The most important properties of the model were: two-dimensional, linear elastic, meshed with quadratic 8-

node PLANE183 elements, refined mesh around the crack tip, plane strain conditions. The square root singularity of the stress at the crack tip was emphasized by means of using shifted mid-side in the first row of elements. The elastic material constants corresponded to the PMMA material teste experimentally, i.e. Young's modulus $E = 2900$ MPa and Poisson's ratio $\nu = 0.35$.

From the FE solution of the problem, the displacement field of the nodes at the distance of 1 mm from the crack tip was taken to apply the ODM. The ODM procedure was programmed in Wolfram Mathematica software (Wolfram Mathematica, 2018). The coefficients of higher-order terms have been obtained as the result.

The tangential stress could be then reconstructed at various distances considering various numbers (between 1 and 10) of initial higher-order terms of the WE, see the range where the tangential stress was investigated in Fig. 12.1b. In order to find the angle of the further crack propagation, the angle where the tangential stress reaches its maximum had to be found. The influence of the selected critical distance as well as number of initial WE terms is discussed in the following section.

12.5. Results

In Fig. 12.2 the tangential stress distribution approximated by means of Eq. 12.1. and taking into account 1, 2, 3, 6 and 10 initial terms of the WE. The stress is reconstructed in front of the crack tip according to Fig. 12.1b. Comparison between the specimens with various widths ($W = 30$ (CT), W = 60, 90 and 150 mm (DCB)) as well as between the results calculated at various distances from the crack tip ($r_c = 0.1$, 1, and 5 mm) can be seen.

The results presented in Fig. 12.2 can be commented with regard to the choice of the proper critical distance in the following way. If the critical distance is chosen very small, the singularity of the first singular term is dominant, and these results differ significantly from the ones obtained by means of the WE assuming more terms. This is more evident for specimens with larger widths.

When the point of view of the angle of the further crack propagation is considered (using the generalized MTS criterion), it can be seen in Fig. 12.2 that when the criterion is applied at a very small distance (r_c = 0.1 mm), the prediction of the propagation angle would be the same independently on the specimen width – i.e. the crack should propagate along its original plane. The same results were observed when the classical one-parameter form of the MTS criterion is assumed ($NM = 1$ in Fig. 12.2). But the experiments performed on PMMA CT and especially DCB specimens say something else (Ayatollahi *et* *al.*, 2016): when the specimen width is 60 mm and larger, the crack kinks from its original direction in spite the fact that it is loaded under pure mode I. Thus, the generalized (multi-parameter) MTS criterion instead of the classical one-parameter should be used and furthermore, the critical distance where the criterion is applied must be large enough.

Although there have been published several recommendations how to estimate this parameter r_c (Seweryn and Lukaszewicz, 2002; Sih and Ho, 1991; Sumsle, 2008), a really universal and reliable instruction does not exist yet. Therefore, several values of the critical distance are applied, tested and based on the comparison between the numerical an experimental results discussed.

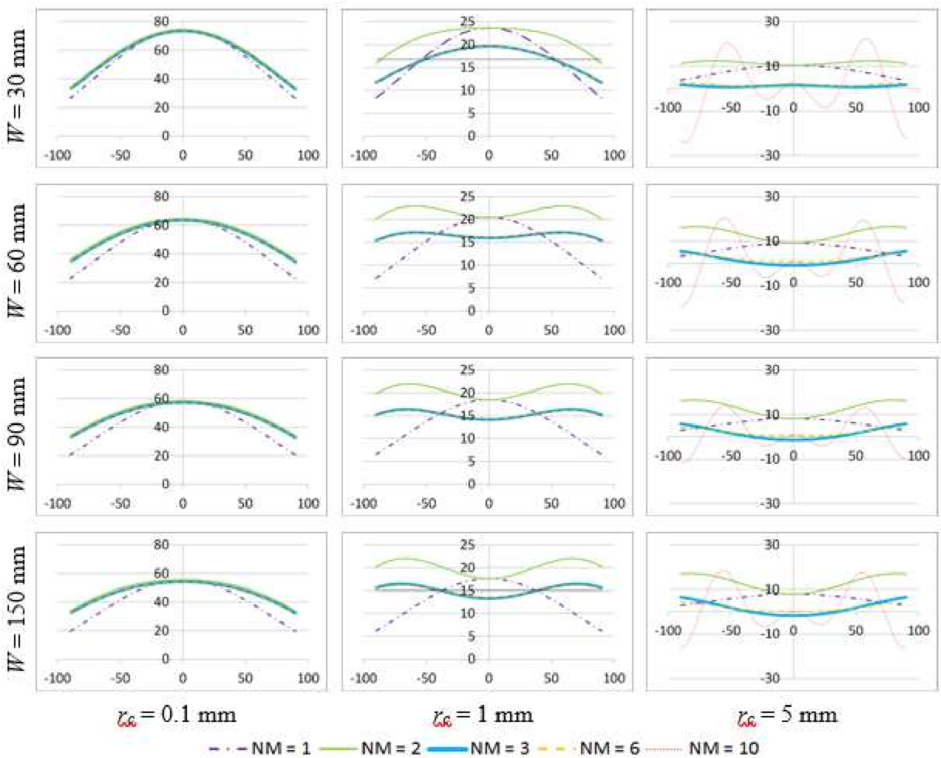


Fig. 12.2. Dependence of the tangential stress distribution (vertical axes in [MPa]) ahead of the crack tip on the angle θ between −90° and 90° (horizontal axes) for various critical distances r_c = 0.1, 1 and 5 mm approximated via WE assuming 1, 2, 3, 6 and 10 initial higher-order terms. Each row corresponds to the cracked specimen with different width: W = 30 (CT), W = 60, 90 and 150 mm (DCB)

In Fig. 12.3 the dependence of the initial crack propagation angle determined by means of the generalized (multi-parameter) MTS criterion considering various numbers of the initial terms on the critical distance where the criterion is applied can be seen for cracked specimens with different widths *W* = 30, 60, 90 and 150 mm.

From the results presented in Fig. 12.3 the following conclusions can be highlighted:

- When the crack propagation in the CT specimen is investigated, it is found out that the crack will propagate in its original direction (no kinking occurs) independently on the distance where the generalized MTS criterion is applied as well as on the number of the initial WE terms considered; this fully agrees with the experimental observations (Ayatollahi *et al.*, 2016).

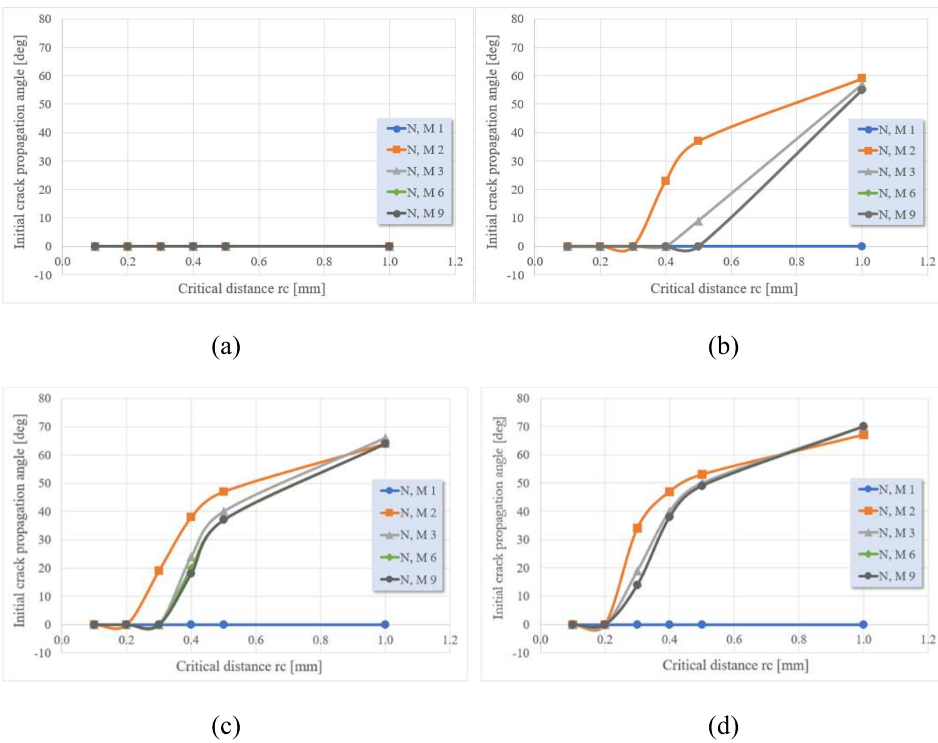


Fig. 12.3. How the initial crack propagation angle depends on the critical distance where the generalized (multi-parameter) MTS criterion (considering 1, 2, 3, 6 and 9 initial terms of the WE) is applied: (a) CT specimen, *W* = 30 mm, (b) DCB specimen, *W* = 60 mm, (c) DCB specimen, *W* = 90 mm, (d) DCB specimen, *W* = 150 mm

- When the initial crack propagation angle in DCB specimens is investigated, various results of the kink angle can be obtained in dependence on the choice of the critical distance and on the number of the WE terms assumed, see the following discussion.
- When the crack propagation in the DCB specimens with larger width (higher geometric constraint) is investigated, it is found out that the classical (one-parameter) MTS criterion is not able to describe the crack deflection; it predicts the straightforward crack propagation for all the DCB configurations considered despite the experimental results.
- Therefore, the multi-parameter (generalized) form of the MTS criterion is to be recommended for this kind of specimens to get better results.
- The initial kink angle calculated by means of the multi-parameter MTS criterion considering 3 and more WE terms does not change significantly when more WE terms are considered.
- When the crack propagation angle is determined from the stress distribution very close to the crack tip, it seems that the crack will propagate straight ahead for all specimen widths, which is inconsistent with the experiments.
- Therefore, rather larger critical distance can be recommended.
- When the experimental values of the initial crack propagation angle are compared to the dependences obtained numerically, the generalized MTS criterion considering initial 3 terms of the WE brings good results for the critical distance of 0.6 mm for 150 mm wide DCB specimens and critical distance of 0.35 mm for 90 mm wide DCB specimens.

12.6. Conclusions

Within this work, the importance of the multi-parameter (generalized) fracture mechanics is emphasized. It is shown that the classical one-parameter fracture mechanics is not able to describe the crack kinking when the crack propagation in the specimens with higher geometric constraint is investigated. It is also proved that the proper choice of the critical distance where the generalized fracture criterion is applied is crucial. All the conclusions are made based on the comparison between the numerical analysis and experimental campaign. It is recommended to consider at least three terms of the WE when the crack propagation in the DCB specimens with larger width shall be investigated. The critical distance between 0.35 and 0.6 mm seems to be suitable for the specimens with the defined geometry and made of the material under the study.

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