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Deflection of an eccentric crack under mixed-mode conditions in an SCB specimen

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Abstract: Crack propagation under mixed-mode (I + II) conditions has been investigated in a semi-circular disc where various levels of mixed-mode can be achieved by means of different geometry configurations. The research has been performed on a novel cementitious material, alkali-activated concrete. Its main advantage is that it is environment-friendly. On the other hand, its fracture mechanical properties, as of yet, have not been described sufficiently. Therefore, a fracture analysis has been performed. The crack deflection under three-point bending conditions has been investigated numerically as well as experimentally. The numerical approach is based on a combination of the common finite element analysis and a multi-parameter form of the maximum tangential stress criterion. This generalized method is suitable especially for materials with specific (elasto-plastic, quasi-brittle etc.) fracture behaviour. The over-deterministic method together with the Williams expansion is applied to approximate selected stress tensor components around the crack tip. In this work, the influence of the eccentric crack is also discussed. In the conclusions, several recommendations about using single-parameter/multi-parameter fracture mechanics are stated.

Keywords: surface cracks, finite element analysis, stress, concrete

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Introduction

Fracture mechanics is crucial when the fracture response of structures made of different materials are assessed. It is an area of constant development because more and more novel materials with tailored properties are emerging. The requirements of materials used for a structure can be different, one such requirement is to be environment-friendly. Such a material has been selected for this study, specifically, the fracture behaviour of a crack in a specimen made of an alkali-activated concrete has been investigated. Use of new materials is connected to the demand for new fracture mechanical concepts suitable for materials with non-brittle-like failures.

Recent efforts have included more accurate approximation of crack-tip stress and/or displacement fields that are crucial for additional more comprehensive fracture analyses. The multi-parameter/generalized approach proved to be effective (Du & Hancock, 1991; Karihaloo, 1999; Berto & Lazzarin, 2010). This concept is especially useful when a fracture occurs further away from the crack tip, and the classical single-parameter (singular) solution is not valid. The Williams expansion (Williams, 1957) derived for approximation of the crack-tip stress/displacement field can be utilized assuming several initial terms of the power series.

Furthermore, a new fracture criterions have been derived, for example, in (Smith et al., 2001; Hou et al., 2016) where an advanced form of the common maximum tangential stress criterion for estimation of further crack propagation is applied considering higher-order terms of the crack-tip stress field approximation. In Ayatollahi & Nejati (2011a, 2011b) the higher-order terms coefficients for cracked and V-notched specimens are computed by using a numerical procedure called the over-deterministic method (ODM). The method is used also within this paper. They also proved that the non-singular terms of the series play an important role in the derivation of crack-tip stress approximation. Several recent works from the authors are devoted to the investigation of the significance of the truncated form of the power expansion (Malíková, 2015; Malíková & Veselý, 2015, 2017; Veselý et al., 2015).

In this work, a numerical analysis is performed on a semi-circular disc under three-point bending (SCB). The crack inclination angle together with further geometrical parameters ensures various levels of (I + II) mixed-mode conditions on the crack-tip. The initial crack propagation angle is investigated by means of the generalized MTS criterion as well as measured experimentally. Results obtained on configurations with an eccentric crack are compared to results for a crack going from the center of the specimen.

1. Approximation of the crack-tip stress field

As stated above, the stress tensor components were firstly expressed as a series expansion by Williams (Williams, 1957). The relations were derived for a homogeneous elastic isotropic cracked body subjected to an arbitrary remote loading and can be written for the stress tensor components and displacement vector components, respectively as follows:

$$\sigma_{ij} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n,\theta) A_n + \sum_{m=1}^{\infty} \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m,\theta) B_m \quad (1)$$

$$u_i = \sum_{n=0}^{\infty} r^{\frac{n}{2}} f_i(n,\theta,E,\nu) A_n + \sum_{m=0}^{\infty} r^{\frac{m}{2}} g_i(m,\theta,E,\nu) B_m \quad (2)$$

where:

i, j – stress tensor/displacement vector components indexes: $i, j \in \{x, y\}$,
m, n – indexes of the Williams expansion series,
r, θ – polar coordinates with the centre of the system at the crack tip,
f_{ij}, g_{ij} – known stress functions corresponding to the loading mode I and II, respectively,
f_i, g_i – known displacement functions corresponding to the loading mode I and II, respectively,
E – Young's modulus,
ν – Poisson's ratio,
A_n, B_m – unknown coefficients of the higher-order terms of the Williams expansion (WE).

The Williams expansion derived for the displacement vector components is the basic equation for application of the ODM.

2. Over-deterministic method

So far, several methods have been derived for estimation of the WE coefficients (Tong et al., 1973; Karihaloo & Xiao, 2001; Xiao et al., 2004; Su & Fok, 2007; Ayatollahi & Nejati, 2011a). Among methods such as the boundary collocation method, hybrid crack element method etc., the over-deterministic method seems to be the easiest and therefore it was applied within this research. The main advantage is its requirement on the basic outputs of a common finite element analysis: only displacements of a selected set of nodes around the crack tip need to be known (together with the polar coordinates of each node). By means of solving the system of equations, the coefficients of the higher-order terms of the WE are obtained. To satisfy the condition of the over-determined system, it has to hold $2k > N + M + 2$, where k represents the number of nodes selected around the crack tip, N and M represent the numbers of mode I and mode II terms, respectively. See more details on the ODM for instance in (Šestáková, 2013; Růžička et al., 2017).

3. Maximum tangential stress criterion

The principle of searching for the initial crack propagation angle consists in the application of the maximum tangential stress criterion, particularly its generalized (multi-parameter) form. Thanks to the knowledge of the coefficients of the WE, an arbitrary stress tensor component can be approximated by means of the truncated WE series considering a selected number of the initial terms.

The idea of the MTS criterion (Erdogan & Sih, 1963) is based on looking for the direction, where the tangential stress reaches its maximum. This condition can be written mathematically as:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \sigma_{ij} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n,\theta) + \sum_{n=1}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m,\theta)$$

and $i,j \in \{x,y\} \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \{\sigma_{xx}\ \sigma_{yy}\ \tau_{xy}\} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} \cdot$

$$\Big\{\Big[2 + (-1)^n + \frac{n}{2}\Big] \cos\cos\Big(\frac{n}{2} - 1\Big)\theta - \Big(\frac{n}{2} - 1\Big) \cos\cos\Big(\frac{n}{2} - 3\Big)\theta \ \Big[2 - (-1)^n - \frac{n}{2}\Big] \cos\cos\Big(\frac{n}{2} - 1\Big)\theta + \Big(\frac{n}{2} - 1\Big) \cos\cos\Big(\frac{n}{2} - 3\Big)\theta \ - \Big[(-1)^n + \frac{n}{2}\Big] \sin\sin\Big(\frac{n}{2} - 1\Big)\theta + \Big(\frac{n}{2} - 1\Big) \sin\sin\Big(\frac{n}{2} - 3\Big)\theta \ \Big\} + + \sum_{m=1}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}-1} \cdot$$

$$\Big\{\Big[2 - (-1)^m + \frac{m}{2}\Big] \sin\sin\Big(\frac{m}{2} - 1\Big)\theta - \Big(\frac{m}{2} - 1\Big) \sin\sin\Big(\frac{m}{2} - 3\Big)\theta \ \Big[2 + (-1)^m - \frac{m}{2}\Big] \sin\sin\Big(\frac{m}{2} - 1\Big)\theta + \Big(\frac{m}{2} - 1\Big) \sin\sin\Big(\frac{m}{2} - 3\Big)\theta \ \Big[-(-1)^m + \frac{m}{2}\Big] \cos\cos\Big(\frac{m}{2} - 1\Big)\theta - \Big(\frac{m}{2} - 1\Big) \cos\cos\Big(\frac{m}{2} - 3\Big)\theta \ \Big\} \qquad (3)$$

where:
$\sigma_{\theta\theta}$ – tangential stress,
θ – angle of the polar coordinate system with its centre at the crack tip.

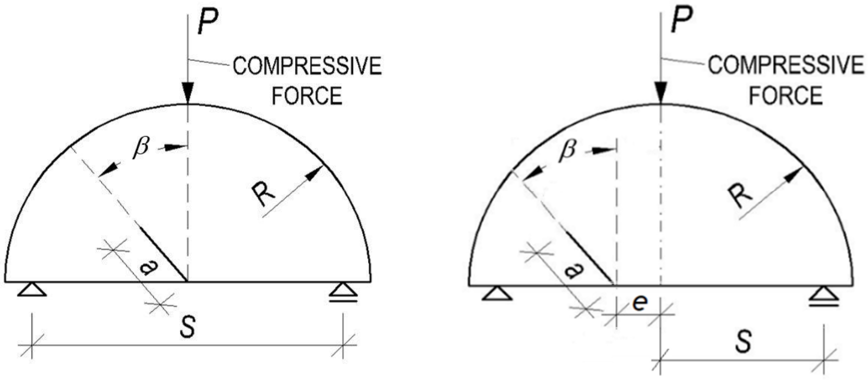
The procedure for searching for tangential stress is then programmed in the Wolfram Mathematica software (Wolfram Mathematica Documentation Center, 2018).

It should be noted that the multi-parameter fracture mechanics concept exhibits a new kind of dependence. The MTS criterion is dependent on a length parameter that is often called a critical distance. Recent works (Sih & Ho, 1991; Seweryn & Lukaszewicz, 2002; Susmel & Taylor, 2008) are devoted to investigations on the theory of critical distances (TCD) and recommendations on the proper choice of the critical distance value.

In this work, the critical distance of 2 mm was assumed based on the idea that it should correspond to a characteristic length parameter of the material structure, see the material parameters (such as grain size) in the following section.

4. Specimen geometry, material properties and numerical model

A semi-circular disc with an inclined crack under three-point bending has been selected for the study of the initial crack propagation direction. The advantage is to involve various mixed-mode level conditions by means of different geometrical parameters. Also, configurations with an eccentric crack were considered within the research and compared to basic ones (Fig. 1).



a) central crack b) eccentric crack

Fig. 1. Schema of the semi-circular disc under 3-point bending with an inclined crack: a) configurations with a central crack; b) configurations with an eccentric crack, where: P – loading force, S – half-span between the supports, R – radius of the disc, a – crack length, e – crack eccentricity, β – angle of the inclined crack (*own study*)

Although various combinations of the individual parameters have been assumed for the analysis, in this work only the results for selected geometrical values are presented. For all configurations it holds that $P = 1$ kN, $R = 50$ mm, $S = 80$ mm and $a = 10$ mm. The numerical and experimental analyses were performed for the configurations with: $e = 0$ and $\beta = 30$, 40 and 50° for a central crack and $e = 10$ and 15 mm and $\beta = 50°$ for an eccentric crack, respectively. Thus, K_I/K_{II} ratio (mixed-mode level) ranges from ca 1.5 to 3.1. Note that three real SCB specimens have been tested for each configuration and the results have been treated statistically. For evaluation of the further crack propagation direction via the multi-parameter MTS criterion, critical distances of 0.1, 0.5, 1.0 and 1.5 mm were assumed in order to be able to compare and discuss the resulting dependences.

Based on the geometry presented above, a 2D numerical model has been created. The specimen was modelled by means of quadrilateral 8-node elements PLANE183 in a commercial finite element (FE) code ANSYS. A refined mesh assuming the stress singularity at the crack tip was modelled via KSCON command. To be able to compare the results of the numerical analysis with experimental data, plane stress conditions were defined in the FE model. This assumption respects the evaluation procedure when the values of the crack deflection angles from the laboratory tests were measured optically at the specimen surface at the distance of 2 mm from the crack tip.

Material properties of the numerical model were set up in accordance to the properties of the novel alkali-activated concrete measured on prismatic bars by means of the relevant methods: Young's modulus $E = 35$ MPa, Poisson's ratio $\nu = 0.23$. The alkali-activated slag mortar (AASM) was selected as an environment-friendly material instead of common concrete. This material was manufactured by mixing the ground granulated blast furnace slag from the Czech company (Kotouč

Štramberk, spol. s.r.o) with a Blaine fineness of 400 m^2/kg and standard sand with a maximum grain size of 2 mm. The aggregate-slag ratio (a/s) was 3:1, the water-slag ratio (w/s) was 0.45 and the dose of sodium hydroxide activator was 6% of Na_2O. To maintain the appropriate workability of mortar, the lignosulfonate-based plasticiser was dosed in the amount of 1% by mass of slag.

5. Results and discussion

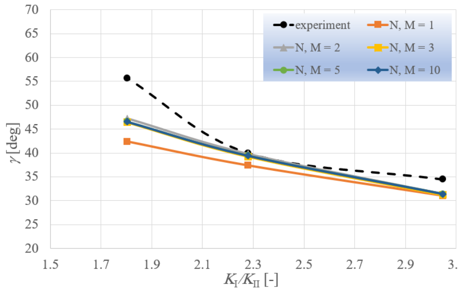
As described above, the crack deflection angle in SCB specimens under various mixed-mode levels has been investigated numerically as well as experimentally. The results obtained for the selected geometry can be seen in Figure 2. Particularly, dependences of the crack deflection angle γ on the K_I/K_{II} ratio observed experimentally (dashed line) and estimated numerically (full lines) considering 1, 2, 3, 5 and 10 initial terms of the Williams expansion in the generalized MTS criterion are displayed. The plots represent the results for the critical distances r_c = 0.1 (a, b), 0.5 (c, d) , 1.0 (e, f) and 1.5 (g, h) mm. The left column of plots represents the data for the SCB specimens with a central crack, whereas the right column represents the data for the SCB specimens with an eccentric crack (see Fig. 1 for more details about the geometry).

It should be noted that with increasing crack inclination angle and/or crack eccentricity loading mode II becomes more relevant (Table 1).

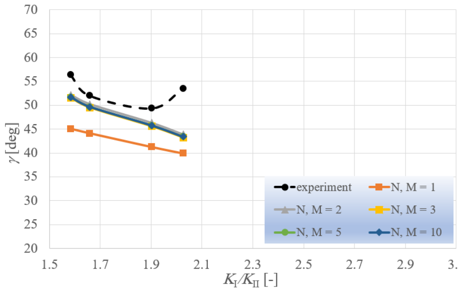
Table 1. Values of the KI/KII ratio (mixed-mode level) for the SCB geometries investigated (*own study*)

	Eccentricity e [mm]	Crack inclination angle β [deg]	Mixed-mode level K_I/K_{II} [–]
Central crack	0	30	3.05
	0	40	2.28
	0	50	1.80
Eccentric crack	10	40	2.03
	10	50	1.66
	15	40	1.90
	15	50	1.58

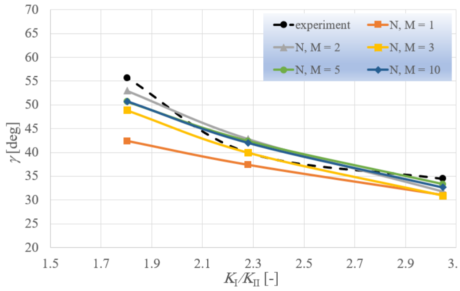
Following trends and conclusions can be observed from the plots presented in Figure 2.



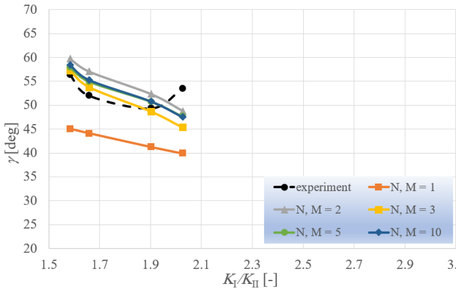
(a) r_c = 0.1 mm, central crack



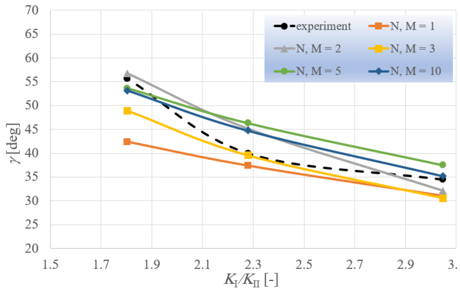
(b) r_c = 0.1 mm, eccentric crack



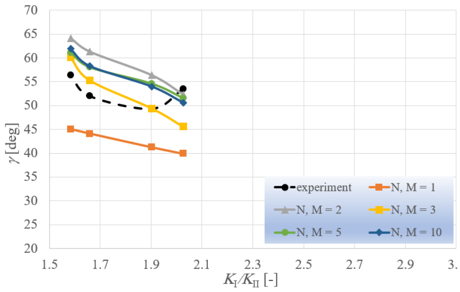
(c) r_c = 0.5 mm, central crack



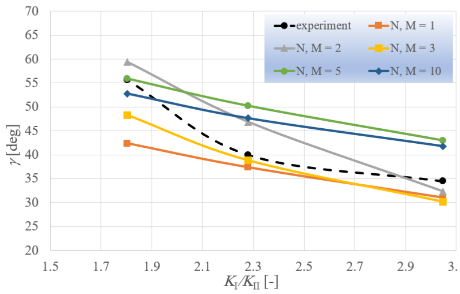
(d) r_c = 0.5 mm, eccentric crack



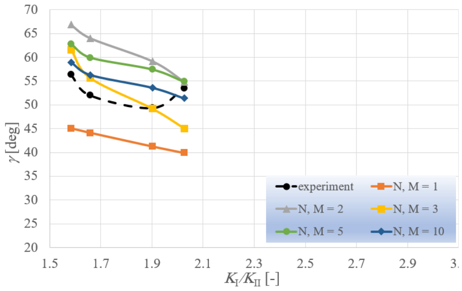
(e) r_c = 1.0 mm, central crack



(f) r_c = 1.0 mm, eccentric crack



(g) r_c = 1.5 mm, central crack



(h) r_c = 1.5 mm, eccentric crack

Fig. 2. Dependence of the crack deflection angle γ on the K_I/K_{II} ratio for the critical distances r_c = 0.1, 0.5, 1.0 and 1.5 mm and 1, 2, 3, 5 and 10 WE terms considered (*own study*)

- The classical single-parameter MTS (SPMTS) criterion is not dependent on the length parameter, therefore its results are the same for each critical distance r_c.
- The classical SPMTS criterion gives the lowest crack deflection angles in comparison to other results from the multi-parameter MTS (MPMTS) criterion.
- The experimental results show that the crack propagates through the SCB specimen under higher deflection angles than the SPMTS criterion predicts.
- Larger differences arise between the crack deflection angles estimated numerically via MPMTS criterion considering various numbers of the initial WE terms at larger critical distances.
- On the other hand, dominant loading mode I decreases the differences between the results obtained via MPMTS assuming various numbers of the initial WE terms.
- When more accurate estimates of the crack deflection angle are obtained, the MPMTS criterion can help significantly, when a proper critical distance as well as a proper number of the initial WE terms considered for the crack-tip stress approximation are chosen.
- For the material under study it seems that the critical distance r_c = 0.5 mm and number of the loading mode I and II WE terms N and M = 3 give the most accurate results in comparison to the experimentally measured data.

Conclusions

The rack deflection angle in an SCB specimen made of a novel environment-friendly material has been investigated experimentally as well as numerically via generalized MTS criterion. Dependence of the crack deflection angle on the mixed-mode level (K_I/K_{II} ratio), critical distance value and number of the Williams expansion terms has been discussed for a central and eccentric crack, respectively. It has been found out that the multi-parameter MTS criterion can bring results similar to experimental data when the relevant parameters are set appropriately. The critical distance r_c = 0.5 mm and N, M = 3 can be recommended for the alkali-activated concrete under study.

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